Wartości p_n dla D podlegających pod rozwiązania główne

W rozdziale poświęconym rozwiązaniom ogólnym, wypisaliśmy równania, które pozwalają na obliczanie wartości p_0, p_1, p_2, $p_3 \ldots\ldots p_n$, kiedy znamy wartość v_2. Z czego wynika, że dla D podlegających pod RG możemy obliczać te wartości.

Dla przypomnienia, chodzi o równania:

"+"		"–"	
(13)	$p_{n+1}^2 + 2tp_n p_{n+1} - (4a^{n+1} + ap_n^2) = 0$	(19)	$p_{n+1}^2 - 2tp_n p_{n+1} - (4a^{n+1} - ap_n^2) = 0$
(14)	$p_{n+1}^2 + 2tp_n p_{n+1} - (2a^{n+1} + ap_n^2) = 0$	(20)	$p_{n+1}^2 - 2tp_n p_{n+1} - (2a^{n+1} - ap_n^2) = 0$
(15)	$p_{n+1}^2 + 2tp_n p_{n+1} - (a^{n+1} + ap_n^2) = 0$	(21)	$p_{n+1}^2 - 2tp_n p_{n+1} - (a^{n+1} - a p_n^2) = 0$
(16)	$p_{n+1}^2 + 2tp_n p_{n+1} + (a^{n+1} - ap_n^2) = 0$	(22)	$p_{n+1}^2 - 2tp_n p_{n+1} + (a^{n+1} + a p_n^2) = 0$
(17)	$p_{n+1}^2 + 2tp_n p_{n+1} + (2a^{n+1} - ap_n^2) = 0$	(23)	$p_{n+1}^2 - 2tp_n p_{n+1} + (2a^{n+1} + ap_n^2) = 0$
(18)	$p_{n+1}^2 + 2tp_n p_{n+1} + (4a^{n+1} - ap_n^2) = 0$	(24)	$p_{n+1}^2 - 2tp_n p_{n+1} + (4a^{n+1} + ap_n^2) = 0$
(27)	$p_{n+1} = a p_{n-1} - 2tp_n$	(28)	$p_{n+1} = 2tp_n - a p_{n-1}$

Przy pomocy równań (13) do (24) będziemy obliczać wartość p_0 i następnie korzystając z równań (27)(28) obliczać wartości p_1 do p_5.

Mamy $p_{n+1} = p_0$ i $p_n = v_2$.

"+"		"–"	
(13)	$p_0^2 + 2tv_2 p_0 - (4 + av_2^2) = 0$	(19)	$p_0^2 - 2tv_2 p_0 - (4 - av_2^2) = 0$
(14)	$p_0^2 + 2tv_2 p_0 - (2 + av_2^2) = 0$	(20)	$p_0^2 - 2tv_2 p_0 - (2 - av_2^2) = 0$
(15)	$p_0^2 + 2tv_2 p_0 - (1 + av_2^2) = 0$	(21)	$p_0^2 - 2tv_2 p_0 - (1 - av_2^2) = 0$
(16)	$p_0^2 + 2tv_2 p_0 + (1 - av_2^2) = 0$	(22)	$p_0^2 - 2tv_2 p_0 + (1 + av_2^2) = 0$
(17)	$p_0^2 + 2tv_2 p_0 + (2 - av_2^2) = 0$	(23)	$p_0^2 - 2tv_2 p_0 + (2 + av_2^2) = 0$
(18)	$p_0^2 + 2tv_2 p_0 + (4 - av_2^2) = 0$	(24)	$p_0^2 - 2tv_2 p_0 + (4 + av_2^2) = 0$

(27)	(28)
$p_1 = av_2 - 2tp_0$	$p_1 = 2tp_0 - av_2$
$p_2 = ap_0 - 2tp_1$	$p_2 = 2tp_1 - ap_0$
$p_3 = ap_1 - 2tp_2$	$p_3 = 2tp_2 - ap_1$
$p_4 = ap_2 - 2tp_3$	$p_4 = 2tp_3 - ap_2$
$p_5 = ap_3 - 2tp_4$	$p_5 = 2tp_4 - ap_3$

W tablicy dla RG podano wartości v_2. Oprócz przypadku $D = t^2 - 1$ dla wszystkich następnych rozwiązań mamy co najmniej dwie wartości dla v_2. W tych przypadkach $v_2 = 1$ lub $v_2 = y$. Ponadto w niektórych przypadkach mamy więcej wartości v_2. Dla każdej wartości v_2 będziemy obliczać p_0, a następnie p_1 do p_5.

Całość obliczeń podzielimy na trzy podrozdziały:

Podrozdział I	$v_2 = 1$	RGI do RGVIII
Podrozdział II	$v_2 = y$	RGI do RGXI
Podrozdział III	v_2 w grupie "±4–1"	

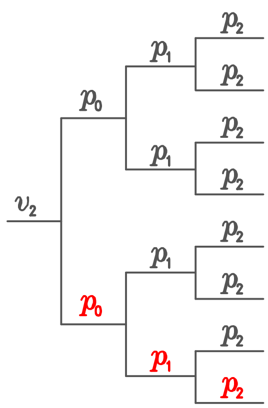
Podrozdział I (RGI do RGVIII)

$v_2 = 1$

W tym przypadku mamy:

"+"		"−"	
(13)	$p_0^2 + 2tp_0 - (4+\alpha) = 0$	(19)	$p_0^2 - 2tp_0 - (4-\alpha) = 0$
(14)	$p_0^2 + 2tp_0 - (2+\alpha) = 0$	(20)	$p_0^2 - 2tp_0 - (2-\alpha) = 0$
(15)	$p_0^2 + 2tp_0 - (1+\alpha) = 0$	(21)	$p_0^2 - 2tp_0 - (1-\alpha) = 0$
(16)	$p_0^2 + 2tp_0 + (1-\alpha) = 0$	(22)	$p_0^2 - 2tp_0 + (1+\alpha) = 0$
(17)	$p_0^2 + 2tp_0 + (2-\alpha) = 0$	(23)	$p_0^2 - 2tp_0 + (2+\alpha) = 0$
(18)	$p_0^2 + 2tp_0 + (4-\alpha) = 0$	(24)	$p_0^2 - 2tp_0 + (4+\alpha) = 0$

W powyższych równaniach dla rozwiązań głównych wyraz wolny zawsze będzie równy 0, a $\sqrt{\triangle} = 2t$. Z dwóch wartości p_0 jedna (mniejsza pod względem wartości bezwzględnej) to zawsze 0 i tą wartość przyjmujemy do dalszych obliczeń, bo chcemy znaleźć rozwiązania najmniejsze z możliwych. Pełne roz–wiązanie otrzymujemy biorąc do dalszych obliczeń obie wartości. Prowadzi to do otrzymania 4–ech tości dla p_1, 8–miu dla p_2, itd..., co graficznie możemy przedstawić jak niżej.



Podobnie jak w rozdziale "O wartościach p_n" będziemy obliczać najmniejsze możliwe wartości p_n (te zaznaczone na czerwono).

Tak więc mamy w tym podrozdziale zawsze:

$$v_2 = 1 \qquad p_0 = 0$$

RGI

D = $t^2 - 1$ $\qquad v_2 = 1 \qquad p_0 = 0 \qquad \alpha = 1$ $\qquad$ znak "−"

(28)

$$p_1 = 2tp_0 - \alpha v_2 = 2t\cdot 0 - 1\cdot 1 = -1$$
$$p_2 = 2tp_1 - \alpha p_0 = 2t(-1) - 1\cdot 0 = -2t$$
$$p_3 = 2tp_2 - \alpha p_1 = 2t(-2t) - 1\cdot(-1) = -(4t^2-1)$$
$$p_4 = 2tp_3 - \alpha p_2 = 2t[-(4t^2-1)] - 1\cdot(-2t) = -4t(2t^2-1)$$
$$p_5 = 2tp_4 - \alpha p_3 = 2t[-4t(2t^2-1)] - 1\cdot[-(4t^2-1)] = -4t^2(4t^2-3) - 1$$

przykłady

D = 24	$t=5$ $\quad a=1$	D = 35	$t=6$ $\quad a=1$
$v_2=1$	$1\cdot1^0+24\cdot1^2=5^2$	$v_2=1$	$1\cdot1^0+35\cdot1^2=6^2$
$p_0=0$	$1\cdot1^1+24\cdot0^2=1^2$	$p_0=0$	$1\cdot1^1+35\cdot0^2=1^2$
$p_1=-1$	$1\cdot1^2+24(-1)^2=5^2$	$p_1=-1$	$1\cdot1^2+35(-1)^2=6^2$
$p_2=-10$	$1\cdot1^3+24(-10)^2=49^2$	$p_2=-12$	$1\cdot1^3+35(-12)^2=71^2$
$p_3=-99$	$1\cdot1^4+24(-99)^2=485^2$	$p_3=-143$	$1\cdot1^4+35(-143)^2=846^2$
$p_4=-980$	$1\cdot1^5+24(-980)^2=4801^2$	$p_4=-1704$	$1\cdot1^5+35(-1704)^2=10081^2$
$p_5=-9701$	$1\cdot1^6+24(-9701)^2=47525^2$	$p_5=-20305$	$1\cdot1^6+35(-20305)^2=120126^2$

RGII

$D=t^2+1 \qquad v_2=1 \qquad p_0=0 \qquad a=1 \qquad$ znak "+"

(27)

$$p_1 = av_2 - 2tp_0 = 1\cdot1-2t\cdot0=1$$
$$p_2 = ap_0 - 2tp_1 = 1\cdot0-2t\cdot1=-2t$$
$$p_3 = ap_1 - 2tp_2 = 1\cdot1-2t(-2t)=4t^2+1$$
$$p_4 = ap_2 - 2tp_3 = 1\cdot(-2t)-2t(4t^2+1) = -4t(2t^2+1)$$
$$p_5 = ap_3 - 2tp_4 = 1\cdot(4t^2+1)-2t[-4t(2t^2+1)] = 4t^2(4t^2+3)+1$$

przykłady

D = 26	$t=5$ $\quad a=1$	D = 37	$t=6$ $\quad a=1$
$v_2=1$	$-1\cdot1^0+26\cdot1^2=5^2$	$v_2=1$	$-1\cdot1^0+37\cdot1^2=6^2$
$p_0=0$	$+1\cdot1^1+26\cdot0^2=1^2$	$p_0=0$	$+1\cdot1^1+37\cdot0^2=1^2$
$p_1=1$	$-1\cdot1^2+26\cdot1^2=5^2$	$p_1=1$	$-1\cdot1^2+37\cdot1^2=6^2$
$p_2=-10$	$+1\cdot1^3+26(-10)^2=51^2$	$p_2=-12$	$+1\cdot1^3+37(-12)^2=73^2$
$p_3=101$	$-1\cdot1^4+26\cdot101^2=515^2$	$p_3=145$	$-1\cdot1^4+37\cdot145^2=882^2$
$p_4=-1020$	$+1\cdot1^5+26(-1020)^2=5201^2$	$p_4=-1752$	$+1\cdot1^5+37(-1752)^2=10657^2$
$p_5=10301$	$-1\cdot1^6+26\cdot10301^2=52525^2$	$p_5=21169$	$-1\cdot1^6+37\cdot21169^2=128766^2$

RGIII

$D=t^2-2 \qquad v_2=1 \qquad p_0=0 \qquad a=2 \qquad$ znak "−"

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot0-2\cdot1=-2$$
$$p_2 = 2tp_1 - ap_0 = 2t(-2)-2\cdot0=-4t$$
$$p_3 = 2tp_2 - ap_1 = 2t(-4t)-2(-2) = -4(2t^2-1)$$
$$p_4 = 2tp_3 - ap_2 = 2t[-4(2t^2-1)]-2(-4t) = -16t(t^2-1)$$
$$p_5 = 2tp_4 - ap_3 = 2t[-16t(t^2-1)]-2[-4(2t^2-1)] = -8[2t^2(2t^2-3)-1]$$

przykłady

D = 23	$t=5$ $\alpha=2$	D = 34	$t=6$ $\alpha=2$
$v_2=1$	$2\cdot2^0+23\cdot1^2=5^2$	$v_2=1$	$2\cdot2^0+34\cdot1^2=6^2$
$p_0=0$	$2\cdot2^1+23\cdot0^2=2^2$	$p_0=0$	$2\cdot2^1+34\cdot0^2=2^2$
$p_1=-2$	$2\cdot2^2+23(-2)^2=10^2$	$p_1=-2$	$2\cdot2^2+34(-2)^2=12^2$
$p_2=-20$	$2\cdot2^3+23(-20)^2=96^2$	$p_2=-24$	$2\cdot2^3+34(-24)^2=140^2$
$p_3=-196$	$2\cdot2^4+23(-196)^2=940^2$	$p_3=-284$	$2\cdot2^4+34(-284)^2=1656^2$
$p_4=-1920$	$2\cdot2^5+23(-1920)^2=9208^2$	$p_4=-3360$	$2\cdot2^5+34(-3360)^2=19592^2$
$p_5=-18808$	$2\cdot2^6+23(-18808)^2=90200^2$	$p_5=-39752$	$2\cdot2^6+34(-39752)^2=231792^2$

RGIV

D = t^2+2 $\quad v_2=1$ $\quad p_0=0$ $\quad \alpha=2$ $\quad$ znak ”+”

(27)

$$p_1=\alpha v_2-2tp_0=2\cdot1-2t\cdot0=2$$
$$p_2=\alpha p_0-2tp_1=2\cdot0-2t\cdot2=-4t$$
$$p_3=\alpha p_1-2tp_2=2\cdot2-2t(-4t)=4(2t^2+1)$$
$$p_4=\alpha p_2-2tp_3=2(-4t)-2t[4(2t^2+1)]=-16t(t^2+1)$$
$$p_5=\alpha p_3-2tp_4=2\cdot4(2t^2+1)-2t[-16t(t^2+1)]=8[2t^2(2t^2+3)+1]$$

przykłady

D = 27	$t=5$ $\alpha=2$	D = 38	$t=6$ $\alpha=2$
$v_2=1$	$-2\cdot2^0+27\cdot1^2=5^2$	$v_2=1$	$-2\cdot2^0+38\cdot1^2=6^2$
$p_0=0$	$+2\cdot2^1+27\cdot0^2=2^2$	$p_0=0$	$+2\cdot2^1+38\cdot0^2=2^2$
$p_1=2$	$-2\cdot2^2+27\cdot2^2=10^2$	$p_1=2$	$-2\cdot2^2+38\cdot2^2=12^2$
$p_2=-20$	$+2\cdot2^3+27(-20)^2=104^2$	$p_2=-24$	$+2\cdot2^3+38(-24)^2=148^2$
$p_3=204$	$-2\cdot2^4+27\cdot204^2=1060^2$	$p_3=292$	$-2\cdot2^4+38\cdot292^2=1800^2$
$p_4=-2080$	$+2\cdot2^5+27(-2080)^2=10808^2$	$p_4=-3552$	$+2\cdot2^5+38(-3552)^2=21896^2$
$p_5=21208$	$-2\cdot2^6+27\cdot21208^2=110200^2$	$p_5=43208$	$-2\cdot2^6+38\cdot43208^2=266352^2$

RGV

D = $(2t_1)^2-4$ $\quad v_2=1$ $\quad p_0=0$ $\quad \alpha=4$ $\quad t=2t_1$ $\quad$ znak ”–”

(28)

$$p_1=2tp_0-\alpha v_2=2t\cdot0-4\cdot1=-4$$
$$p_2=2tp_1-\alpha p_0=2t(-4)-4\cdot0=-8t$$
$$p_3=2tp_2-\alpha p_1=2t(-8t)-4(-4)=-16(t^2-1)$$
$$p_4=2tp_3-\alpha p_2=2t[-16(t^2-1)]-4(-8t)=-32t(t^2-2)$$
$$p_5=2tp_4-\alpha p_3=2t[-32t(t^2-2)]-4[-16(t^2-1)]=-64[t^2(t^2-3)+1]$$

przykłady

D = 32	$t=6$ $\alpha=4$	D = 60	$t=8$ $\alpha=4$
$v_2=1$	$4\cdot4^0+32\cdot1^2=6^2$	$v_2=1$	$4\cdot4^0+60\cdot1^2=8^2$
$p_0=0$	$4\cdot4^1+32\cdot0^2=4^2$	$p_0=0$	$4\cdot4^1+60\cdot0^2=4^2$
$p_1=-4$	$4\cdot4^2+32(-4)^2=24^2$	$p_1=-4$	$4\cdot4^2+60(-4)^2=32^2$
$p_2=-48$	$4\cdot4^3+32(-48)^2=272^2$	$p_2=-64$	$4\cdot4^3+60(-64)^2=496^2$

$p_3 = -560$ $\quad 4\cdot4^4 + 32(-560)^2 = 3168^2$

$p_4 = -6528$ $\quad 4\cdot4^5 + 32(-6528)^2 = 36928^2$

$p_5 = -76096$ $\quad 4\cdot4^6 + 32(-76096)^2 = 430464^2$

$p_3 = -1008$ $\quad 4\cdot4^4 + 60(-1008)^2 = 7808^2$

$p_4 = -15872$ $\quad 4\cdot4^5 + 60(-15872)^2 = 122944^2$

$p_5 = -249920$ $\quad 4\cdot4^6 + 60(-249920)^2 = 1935872^2$

RGVI

$D = (2t_1)^2 + 4 \qquad v_2 = 1 \qquad p_0 = 0 \qquad a = 4 \qquad t = 2t_1$ $\qquad$ znak "+"

(27)

$$p_1 = av_2 - 2tp_0 = 4\cdot1 - 2t\cdot0 = 4$$
$$p_2 = ap_0 - 2tp_1 = 4\cdot0 - 2t\cdot4 = -8t$$
$$p_3 = ap_1 - 2tp_2 = 4\cdot4 - 2t(-8t) = 16(t^2+1)$$
$$p_4 = ap_2 - 2tp_3 = 4(-8t) - 2t[16(t^2+1)] = -32t(t^2+2)$$
$$p_5 = ap_3 - 2tp_4 = 4\cdot16(t^2+1) - 2t[-32t(t^2+2)] = 64[t^2(t^2+3)+1]$$

przykłady

$D = 40 \quad t = 6 \quad a = 4$

$v_2 = 1$	$-4\cdot4^0 + 40\cdot1^2 = 6^2$
$p_0 = 0$	$+4\cdot4^1 + 40\cdot0^2 = 4^2$
$p_1 = 4$	$-4\cdot4^2 + 40\cdot4^2 = 24^2$
$p_2 = -48$	$+4\cdot4^3 + 40(-48)^2 = 304^2$
$p_3 = 592$	$-4\cdot4^4 + 40\cdot592^2 = 3744^2$
$p_4 = -7296$	$+4\cdot4^5 + 40(-7296)^2 = 46144^2$
$p_5 = 89920$	$-4\cdot4^6 + 40\cdot89920^2 = 568704^2$

$D = 68 \quad t = 8 \quad a = 4$

$v_2 = 1$	$-4\cdot4^0 + 68\cdot1^2 = 8^2$
$p_0 = 0$	$+4\cdot4^1 + 68\cdot0^2 = 4^2$
$p_1 = 4$	$-4\cdot4^2 + 68\cdot4^2 = 32^2$
$p_2 = -64$	$+4\cdot4^3 + 68(-64)^2 = 528^2$
$p_3 = 1040$	$-4\cdot4^4 + 68\cdot1040^2 = 8576^2$
$p_4 = -16896$	$+4\cdot4^5 + 68(-16896)^2 = 139328^2$
$p_5 = 274496$	$-4\cdot4^6 + 68\cdot274496^2 = 2263552^2$

RGVII

$D = (2t_1+1)^2 - 4 \qquad v_2 = 1 \qquad p_0 = 0 \qquad t = 2t_1+1 \qquad a = 4$ $\qquad$ znak "−"

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot0 - 4\cdot1 = -4$$
$$p_2 = 2tp_1 - ap_0 = 2t(-4) - 4\cdot0 = -8t$$
$$p_3 = 2tp_2 - ap_1 = 2t(-8t) - 4(-4) = -16(t^2-1)$$
$$p_4 = 2tp_3 - ap_2 = 2t[-16(t^2-1)] - 4(-8t) = -32t(t^2-2)$$
$$p_5 = 2tp_4 - ap_3 = 2t[-32t(t^2-2)] - 4[-16(t^2-1)] = -64[t^2(t^2-3)+1]$$

przykłady

$D = 21 \quad t = 5 \quad a = 4$

$v_2 = 1$	$4\cdot4^0 + 21\cdot1^2 = 5^2$
$p_0 = 0$	$4\cdot4^1 + 21\cdot0^2 = 4^2$
$p_1 = -4$	$4\cdot4^2 + 21(-4)^2 = 20^2$
$p_2 = -40$	$4\cdot4^3 + 21(-40)^2 = 184^2$
$p_3 = -384$	$4\cdot4^4 + 21(-384)^2 = 1760^2$
$p_4 = -3680$	$4\cdot4^5 + 21(-3680)^2 = 16864^2$
$p_5 = -35264$	$4\cdot4^6 + 21(-35264)^2 = 161600^2$

$D = 45 \quad t = 7 \quad a = 4$

$v_2 = 1$	$4\cdot4^0 + 45\cdot1^2 = 7^2$
$p_0 = 0$	$4\cdot4^1 + 45\cdot0^2 = 4^2$
$p_1 = -4$	$4\cdot4^2 + 45(-4)^2 = 28^2$
$p_2 = -56$	$4\cdot4^3 + 45(-56)^2 = 376^2$
$p_3 = -768$	$4\cdot4^4 + 45(-768)^2 = 5152^2$
$p_4 = -10528$	$4\cdot4^5 + 45(-10528)^2 = 70624^2$
$p_5 = -144320$	$4\cdot4^6 + 45(-144320)^2 = 968128^2$

$D = (2t_1+1)^2 + 4 \qquad v_2 = 1 \qquad p_0 = 0 \qquad t = 2t_1+1 \qquad a = 4 \quad$ znak "+"

(27)

$p_1 = av_2 - 2tp_0 = 4\cdot1 - 2t\cdot0 = 4$

$p_2 = ap_0 - 2tp_1 = 4\cdot0 - 2t\cdot4 = -8t$

$p_3 = ap_1 - 2tp_2 = 4\cdot4 - 2t(-8t) = 16(t^2+1)$

$p_4 = ap_2 - 2tp_3 = 4(-8t) - 2t\cdot16(t^2+1) = -32t(t^2+2)$

$p_5 = ap_3 - 2tp_4 = 4\cdot16(t^2+1) - 2t[-32t(t^2+2)] = 64[t^2(t^2+3)+1]$

przykłady

$D = 29 \qquad t = 5 \qquad a = 4$

$v_2 = 1 \qquad -4\cdot4^0 + 29\cdot1^2 = 5^2$

$p_0 = 0 \qquad +4\cdot4^1 + 29\cdot0^2 = 4^2$

$p_1 = 4 \qquad -4\cdot4^2 + 29\cdot4^2 = 20^2$

$p_2 = -40 \qquad +4\cdot4^3 + 29(-40)^2 = 216^2$

$p_3 = 416 \qquad -4\cdot4^4 + 29\cdot416^2 = 2240^2$

$p_4 = -4320 \qquad +4\cdot4^5 + 29(-4320)^2 = 23264^2$

$p_5 = 44864 \qquad -4\cdot4^6 + 29\cdot44864^2 = 241600^2$

$D = 53 \qquad t = 7 \qquad a = 4$

$v_2 = 1 \qquad -4\cdot4^0 + 53\cdot1^2 = 7^2$

$p_0 = 0 \qquad +4\cdot4^1 + 53\cdot0^2 = 4^2$

$p_1 = 4 \qquad -4\cdot4^2 + 53\cdot4^2 = 28^2$

$p_2 = -56 \qquad +4\cdot4^3 + 53(-56)^2 = 408^2$

$p_3 = 800 \qquad -4\cdot4^4 + 53\cdot800^2 = 5824^2$

$p_4 = -11424 \qquad +4\cdot4^5 + 53(-11424)^2 = 83168^2$

$p_5 = 163136 \qquad -4\cdot4^6 + 53\cdot163136^2 = 1187648^2$

Dotychczasowe wyniki możemy przedstawić w formie tabeli jak niżej:

Rozwiązania główne RGI–VIII ($v_2 = 1, \quad p_0 = 0$)

D	v_2	p_0	p_1	p_2	p_3	p_4	p_5	gr.
t^2-1	+ 1	+ 0	+ -1	+ $-2t$	+ $-(4t^2-1)$	+ $-4t(2t^2-1)$	+ $-4t^2(4t^2-3)-1$	I
t^2+1	− 1	+ 0	− 1	+ $-2t$	− $4t^2+1$	+ $-4t(2t^2+1)$	− $4t^2(4t^2+3)+1$	II
t^2-2	+ 1	+ 0	+ -2	+ $-4t$	+ $-4(2t^2-1)$	+ $-16t(t^2-1)$	+ $-8[2t^2(2t^2-3)-1]$	III
t^2+2	− 1	+ 0	− 2	+ $-4t$	− $4(2t^2+1)$	+ $-16t(t^2+1)$	− $8[2t^2(2t^2+3)+1]$	IV
$(2t_1)^2-4$	+ 1	+ 0	+ -4	+ $-8t$	+ $-16(t^2-1)$	+ $-32t(t^2-2)$	+ $-64[t^2(t^2-3)+1]$	V
$(2t_1)^2+4$	− 1	+ 0	− 4	+ $-8t$	− $16(t^2+1)$	+ $-32t(t^2+2)$	− $64[t^2(t^2+3)+1]$	VI
$(2t_1+1)^2-4$	+ 1	+ 0	+ -4	+ $-8t$	+ $-16(t^2-1)$	+ $-32t(t^2-2)$	+ $-64[t^2(t^2-3)+1]$	VII
$(2t_1+1)^2+4$	− 1	+ 0	− 4	+ $-8t$	− $16(t^2+1)$	+ $-32t(t^2+2)$	− $64[t^2(t^2+3)+1]$	VIII

Znaki "+" lub "−" nad poszczególnymi rozwiązaniami mówią o znaku przed wyrażeniem $1,2,4a^n$.

Podrozdział II (RGI do RGXI)

$$v_2 = y$$

W tym podrozdziale traktujemy wszystkie D podlegające pod RGI do RGXI jak liczby z grupy "+1". Co oznacza, że będziemy korzystać z równań (15)(16) dla $D = t^2 + a$ i (21)(22) dla $D = t^2 - a$ na obliczenie wartości p_0 i następnie wartości $p_1, p_2, p_3 \dots$ z wzorów (27)(28).

"+"	"−"
(15) $p_{n+1}^2 + 2tp_np_{n+1} - (a^{n+1} + ap_n^2) = 0$	(21) $p_{n+1}^2 - 2tp_np_{n+1} - (a^{n+1} - ap_n^2) = 0$
(16) $p_{n+1}^2 + 2tp_np_{n+1} + (a^{n+1} - ap_n^2) = 0$	(22) $p_{n+1}^2 - 2tp_np_{n+1} + (a^{n+1} + ap_n^2) = 0$
(27) $p_{n+1} = ap_{n-1} - 2tp_n$	(28) $p_{n+1} = 2tp_n - ap_{n-1}$

Mamy $p_{n+1} = p_0$ i $p_n = v_2$.

(15) $p_0^2 + 2tv_2p_0 - (1 + av_2^2) = 0$	(21) $p_0^2 - 2tv_2p_0 - (1 - av_2^2) = 0$
(16) $p_0^2 + 2tv_2p_0 + (1 - av_2^2) = 0$	(22) $p_0^2 - 2tv_2p_0 + (1 + av_2^2) = 0$

Wartości v_2 bierzemy z tabeli dla RG (str21).

RGI

$D = t^2 - 1$ $v_2 = y = 1$ $a = 1$ znak "−"

(21) $p_0^2 - 2tv_2p_0 - (1 - av_2^2) = 0$ $p_0^2 - 2tp_0 - (1 - 1) = 0$ $\sqrt{\triangle} = 2t$

(22) $p_0^2 - 2tv_2p_0 + (1 + av_2^2) = 0$ $p_0^2 - 2tp_0 + (1 + 1) = 0$ $\sqrt{\triangle} = \sqrt{4t^2 - 8}$ odpada

$$p_0 = \frac{2t + 2t}{2} = 2t \qquad p_0 = \frac{2t - 2t}{2} = 0$$

Mamy więc:

$$v_2 = y = 1 \qquad p_0 = 0$$

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot 0 - 1\cdot 1 = -1$$
$$p_2 = 2tp_1 - ap_0 = 2t(-1) - 1\cdot 0 = -2t$$
$$p_3 = 2tp_2 - ap_1 = 2t(-2t) - 1\cdot(-1) = -(4t^2 - 1)$$
$$p_4 = 2tp_3 - ap_2 = 2t[-(4t^2 - 1)] - 1\cdot(-2t) = -4t(2t^2 - 1)$$
$$p_5 = 2tp_4 - ap_3 = 2t[-4t(2t^2 - 1)] - 1\cdot[-(4t^2 - 1)] = -4t^2(4t^2 - 3) - 1$$

Te wyniki są takie same jak w podrozdziale I, co wynika z faktu, że dla $D = t^2 - 1$ mamy $v_2 = 1 = y$.

przykłady

$D = 24 = 5^2 - 1$ $t = 5$ $a = 1$		$D = 35 = 6^2 - 1$ $t = 6$ $a = 1$	
$v_2 = y = 1$	$1\cdot 1^0 + 24\cdot 1^2 = 5^2$	$v_2 = y = 1$	$1\cdot 1^0 + 35\cdot 1^2 = 6^2$
$p_0 = 0$	$1\cdot 1^1 + 24\cdot 0^2 = 1^2$	$p_0 = 0$	$1\cdot 1^1 + 35\cdot 0^2 = 1^2$
$p_1 = -1$	$1\cdot 1^2 + 24(-1)^2 = 5^2$	$p_1 = -1$	$1\cdot 1^2 + 35(-1)^2 = 6^2$

$p_2=-10$	$1\cdot1^3+24(-10)^2=49^2$	$p_2=-12$	$1\cdot1^3+35(-12)^2=71^2$
$p_3=-99$	$1\cdot1^4+24(-99)^2=485^2$	$p_3=-143$	$1\cdot1^4+35(-143)^2=846^2$
$p_4=-980$	$1\cdot1^5+24(-980)^2=4801^2$	$p_4=-1704$	$1\cdot1^5+35(-1704)^2=10081^2$
$p_5=-9701$	$1\cdot1^6+24(-9701)^2=47525^2$	$p_5=-20305$	$1\cdot1^6+35(-20305)^2=120126^2$

RGII

$D=t^2+1 \qquad v_2=y=2t \qquad a=1 \qquad$ znak "+"

(15) $p_0^2+2tv_2p_0-(1+av_2^2)=0 \qquad p_0^2+4t^2p_0-(1+4t^2)=0 \qquad \sqrt{\triangle}=2(2t^2+1)$

(16) $p_0^2+2tv_2p_0+(1-av_2^2)=0 \qquad p_0^2+4t^2p_0+(1-4t^2)=0 \qquad \sqrt{\triangle}=\sqrt{4(2t^2+1)^2-8}$ odpada

$$p_0=\frac{-4t^2+2(2t^2+1)}{2}=1 \qquad p_0=\frac{-4t^2-2(2t^2+1)}{2}=-(4t^2+1)$$

Mamy więc:

$$v_2=y=2t \qquad p_0=1$$

(27)

$p_1=av_2-2tp_0=1\cdot2t-2t\cdot1=0$

$p_2=ap_0-2tp_1=1\cdot1-2t\cdot0=1$

$p_3=ap_1-2tp_2=1\cdot0-2t\cdot1=-2t$

$p_4=ap_2-2tp_3=1\cdot1-2t(-2t)=4t^2+1$

$p_5=ap_3-2tp_4=1\cdot(-2t)-2t(4t^2+1)=-4t(2t^2+1)$

przykłady

$D=26=5^2+1 \qquad t=5 \qquad a=1$ | $D=37=6^2+1 \qquad t=6 \qquad a=1$

$v_2=y=10$	$+1\cdot1^0+26\cdot10^2=51^2$	$v_2=y=12$	$+1\cdot1^0+37\cdot12^2=73^2$
$p_0=1$	$-1\cdot1^1+26\cdot1^2=5^2$	$p_0=1$	$-1\cdot1^1+37\cdot1^2=6^2$
$p_1=0$	$+1\cdot1^2+26\cdot0^2=1^2$	$p_1=0$	$+1\cdot1^2+37\cdot0^2=1^2$
$p_2=1$	$-1\cdot1^3+26\cdot1^2=5^2$	$p_2=1$	$-1\cdot1^3+37\cdot1^2=6^2$
$p_3=-10$	$+1\cdot1^4+26(-10)^2=51^2$	$p_3=-12$	$+1\cdot1^4+37(-12)^2=73^2$
$p_4=101$	$-1\cdot1^5+26\cdot101^2=515^2$	$p_4=145$	$-1\cdot1^5+37\cdot145^2=882^2$
$p_5=-1020$	$+1\cdot1^6+26(-1020)^2=5201^2$	$p_5=-1752$	$+1\cdot1^6+37(-1752)^2=10657^2$

RGIII

$D=t^2-2 \qquad v_2=y=t \qquad a=2 \qquad$ znak "−"

(21) $p_0^2-2tv_2p_0-(1-av_2^2)=0 \qquad p_0^2-2t^2p_0-(1-2t^2)=0 \qquad \sqrt{\triangle}=2(t^2-1)$

(22) $p_0^2-2tv_2p_0+(1+av_2^2)=0 \qquad p_0^2-2t^2p_0+(1+2t^2)=0 \qquad \sqrt{\triangle}=\sqrt{4(t^2-1)^2-8}$ odpada

$$p_0=\frac{2t^2+2(t^2-1)}{2}=2t^2-1 \qquad p_0=\frac{2t^2-2(t^2-1)}{2}=1$$

Mamy więc:

$$v_2=y=t \qquad p_0=1$$

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot 1 - 2t = 0$$
$$p_2 = 2tp_1 - ap_0 = 2t\cdot 0 - 2\cdot 1 = -2$$
$$p_3 = 2tp_2 - ap_1 = 2t(-2) - 2\cdot 0 = -4t$$
$$p_4 = 2tp_3 - ap_2 = 2t(-4t) - 2(-2) = -4(2t^2-1)$$
$$p_5 = 2tp_4 - ap_3 = 2t[-4(2t^2-1)] - 2(-4t) = -16t(t^2-1)$$

przykłady

$D = 23 = 5^2 - 2 \quad t = 5 \quad a = 2$

$v_2 = y = 5$	$1\cdot 2^0 + 23\cdot 5^2 = 24^2$
$p_0 = 1$	$1\cdot 2^1 + 23\cdot 1^2 = 5^2$
$p_1 = 0$	$1\cdot 2^2 + 23\cdot 0^2 = 2^2$
$p_2 = -2$	$1\cdot 2^3 + 23(-2)^2 = 10^2$
$p_3 = -20$	$1\cdot 2^4 + 23(-20)^2 = 96^2$
$p_4 = -196$	$1\cdot 2^5 + 23(-196)^2 = 940^2$
$p_5 = -1920$	$1\cdot 2^6 + 23(-1920)^2 = 9208^2$

$D = 34 = 6^2 - 2 \quad t = 6 \quad a = 2$

$v_2 = y = 6$	$1\cdot 2^0 + 34\cdot 6^2 = 35^2$
$p_0 = 1$	$1\cdot 2^1 + 34\cdot 1^2 = 6^2$
$p_1 = 0$	$1\cdot 2^2 + 34\cdot 0^2 = 2^2$
$p_2 = -2$	$1\cdot 2^3 + 34(-2)^2 = 12^2$
$p_3 = -24$	$1\cdot 2^4 + 34(-24)^2 = 140^2$
$p_4 = -284$	$1\cdot 2^5 + 34(-284)^2 = 1656^2$
$p_5 = -3360$	$1\cdot 2^6 + 34(-3360)^2 = 19592^2$

RGIV

$D = t^2 + 2 \quad v_2 = y = t \quad a = 2 \quad$ znak "+"

(15) $p_0^2 + 2tv_2p_0 - (1 + av_2^2) = 0 \quad p_0^2 + 2t^2p_0 - (1 + 2t^2) = 0 \quad \sqrt{\triangle} = 2(t^2+1)$

(16) $p_0^2 + 2tv_2p_0 + (1 - av_2^2) = 0 \quad p_0^2 + 2t^2p_0 + (1 - 2t^2) = 0 \quad \sqrt{\triangle} = \sqrt{4(t^2+1)^2 - 8}$ odpada

$$p_0 = \frac{-2t^2 + 2(t^2+1)}{2} = 1 \qquad p_0 = \frac{-2t^2 - 2(t^2+1)}{2} = -(2t^2+1)$$

Mamy więc: $\quad v_2 = y = t \quad p_0 = 1$

(27)

$$p_1 = av_2 - 2tp_0 = 2t - 2t\cdot 1 = 0$$
$$p_2 = ap_0 - 2tp_1 = 2\cdot 1 - 2t\cdot 0 = 2$$
$$p_3 = ap_1 - 2tp_2 = 2\cdot 0 - 2t\cdot 2 = -4t$$
$$p_4 = ap_2 - 2tp_3 = 2\cdot 2 - 2t(-4t) = 4(2t^2+1)$$
$$p_5 = ap_3 - 2tp_4 = 2(-4t) - 2t[4(2t^2+1)] = -16t(t^2+1)$$

przykłady

$D = 27 = 5^2 + 2 \quad t = 5 \quad a = 2$

$v_2 = y = 5$	$+1\cdot 2^0 + 27\cdot 5^2 = 26^2$
$p_0 = 1$	$-1\cdot 2^1 + 27\cdot 1^2 = 5^2$
$p_1 = 0$	$+1\cdot 2^2 + 27\cdot 0^2 = 2^2$
$p_2 = 2$	$-1\cdot 2^3 + 27\cdot 2^2 = 10^2$
$p_3 = -20$	$+1\cdot 2^4 + 27(-20)^2 = 104^2$
$p_4 = 204$	$-1\cdot 2^5 + 27\cdot 204^2 = 1060^2$
$p_5 = -2080$	$+1\cdot 2^6 + 27(-2080)^2 = 10808^2$

$D = 38 = 6^2 + 2 \quad t = 6 \quad a = 2$

$v_2 = y = 6$	$+1\cdot 2^0 + 38\cdot 6^2 = 37^2$
$p_0 = 1$	$-1\cdot 2^1 + 38\cdot 1^2 = 6^2$
$p_1 = 0$	$+1\cdot 2^2 + 38\cdot 0^2 = 2^2$
$p_2 = 2$	$-1\cdot 2^3 + 38\cdot 2^2 = 12^2$
$p_3 = -24$	$+1\cdot 2^4 + 38(-24)^2 = 148^2$
$p_4 = 292$	$-1\cdot 2^5 + 38\cdot 292^2 = 1800^2$
$p_5 = -3552$	$+1\cdot 2^6 + 38(-3552)^2 = 21896^2$

RGV

$D = (2t_1)^2 - 4 \qquad v_2 = y = \frac{t}{2} \qquad v_1 = 1 \quad t = 2t_1 \quad a = 4 \qquad$ znak "–"

(21) $p_0^2 - 2tv_2p_0 - (1 - av_2^2) = 0 \qquad p_0^2 - t^2p_0 - (1 - t^2) = 0 \qquad \sqrt{\triangle} = t^2 - 2$

(22) $p_0^2 - 2tv_2p_0 + (1 + av_2^2) = 0 \qquad p_0^2 - t^2p_0 + (1 + t^2) = 0 \qquad \sqrt{\triangle} = \sqrt{(t^2-2)^2 - 8}$ odpada

$$p_0 = \frac{t^2 + (t^2-2)}{2} = t^2 - 1 \qquad p_0 = \frac{t^2 - (t^2-2)}{2} = 1$$

Mamy więc:

$$v_2 = y = \frac{t}{2} \qquad p_0 = 1$$

(28)

$p_1 = 2tp_0 - av_2 = 2t \cdot 1 - 4 \cdot \frac{t}{2} = 0$

$p_2 = 2tp_1 - ap_0 = 2t \cdot 0 - 4 \cdot 1 = -4$

$p_3 = 2tp_2 - ap_1 = 2t(-4) - 4 \cdot 0 = -8t$

$p_4 = 2tp_3 - ap_2 = 2t(-8t) - 4(-4) = -16(t^2-1)$

$p_5 = 2tp_4 - ap_3 = 2t[-16(t^2-1)] - 4(-8t) = -32t(t^2-2)$

przykłady

$D = 32 = 6^2 - 4 \qquad t = 6 \qquad a = 4$

$v_2 = y = 3$	$1 \cdot 4^0 + 32 \cdot 3^2 = 17^2$
$p_0 = 1$	$1 \cdot 4^1 + 32 \cdot 1^2 = 6^2$
$p_1 = 0$	$1 \cdot 4^2 + 32 \cdot 0^2 = 4^2$
$p_2 = -4$	$1 \cdot 4^3 + 32(-4)^2 = 24^2$
$p_3 = -48$	$1 \cdot 4^4 + 32(-48)^2 = 272^2$
$p_4 = -560$	$1 \cdot 4^5 + 32(-560)^2 = 3168^2$
$p_5 = -6528$	$1 \cdot 4^6 + 32(-6528)^2 = 36928^2$

$D = 60 = 8^2 - 4 \qquad t = 8 \qquad a = 4$

$v_2 = y = 4$	$1 \cdot 4^0 + 60 \cdot 4^2 = 31^2$
$p_0 = 1$	$1 \cdot 4^1 + 60 \cdot 1^2 = 8^2$
$p_1 = 0$	$1 \cdot 4^2 + 60 \cdot 0^2 = 4^2$
$p_2 = -4$	$1 \cdot 4^3 + 60(-4)^2 = 32^2$
$p_3 = -64$	$1 \cdot 4^4 + 60(-64)^2 = 496^2$
$p_4 = -1008$	$1 \cdot 4^5 + 60(-1008)^2 = 7808^2$
$p_5 = -15872$	$1 \cdot 4^6 + 60(-15872)^2 = 122944^2$

RGVI

$D = (2t_1)^2 + 4 \qquad v_2 = y = \frac{t}{2} \qquad t = 2t_1 \quad a = 4 \qquad$ znak "+"

(15) $p_0^2 + 2tv_2p_0 - (1 + av_2^2) = 0 \qquad p_0^2 + t^2p_0 - (1 + t^2) = 0 \qquad \sqrt{\triangle} = t^2 + 2$

(16) $p_0^2 + 2tv_2p_0 + (1 - av_2^2) = 0 \qquad p_0^2 + t^2p_0 + (1 - t^2) = 0 \qquad \sqrt{\triangle} = \sqrt{(t^2+2)^2 - 8}$ odpada

$$p_0 = \frac{-t^2 + (t^2+2)}{2} = 1 \qquad p_0 = \frac{-t^2 - (t^2+2)}{2} = -(t^2+1)$$

Mamy więc:

$$v_2 = y = \frac{t}{2} \qquad p_0 = 1$$

(27)

$p_1 = av_2 - 2tp_0 = 4\frac{t}{2} - 2t \cdot 1 = 0$

$p_2 = ap_0 - 2tp_1 = 4 \cdot 1 - 2t \cdot 0 = 4$

$p_3 = ap_1 - 2tp_2 = 4 \cdot 0 - 2t \cdot 4 = -8t$

$p_4 = ap_2 - 2tp_3 = 4 \cdot 4 - 2t(-8t) = 16(t^2+1)$

$p_5 = ap_3 - 2tp_4 = 4(-8t) - 2t \cdot 16(t^2+1) = -32t(t^2+2)$

przykłady

$D = 40 = 6^2 + 4 \quad t = 6 \quad a = 4$

$v_2 = y = 3 \quad +1\cdot 4^0 + 40\cdot 3^2 = 19^2$
$p_0 = 1 \quad -1\cdot 4^1 + 40\cdot 1^2 = 6^2$
$p_1 = 0 \quad +1\cdot 4^2 + 40\cdot 0^2 = 4^2$
$p_2 = 4 \quad -1\cdot 4^3 + 40\cdot 4^2 = 5^2$
$p_3 = -48 \quad +1\cdot 4^4 + 40(-48)^2 = 304^2$
$p_4 = 592 \quad -1\cdot 4^5 + 40\cdot 592^2 = 3744^2$
$p_5 = -7296 \quad +1\cdot 4^6 + 40(-7296)^2 = 46144^2$

$D = 68 = 8^2 + 4 \quad t = 8 \quad a = 4$

$v_2 = y = 4 \quad +1\cdot 4^0 + 68\cdot 4^2 = 33^2$
$p_0 = 1 \quad -1\cdot 4^1 + 68\cdot 1^2 = 8^2$
$p_1 = 0 \quad +1\cdot 4^2 + 68\cdot 0^2 = 4^2$
$p_2 = 4 \quad -1\cdot 4^3 + 68\cdot 4^2 = 32^2$
$p_3 = -64 \quad +1\cdot 4^4 + 68(-64)^2 = 528^2$
$p_4 = 1040 \quad -1\cdot 4^5 + 68\cdot 1040^2 = 8576^2$
$p_5 = -16896 \quad +1\cdot 4^6 + 68(-16896)^2 = 139328^2$

RGVII

$D = (2t_1+1)^2 - 4 \quad v_2 = y = 2t_1(t_1+1) = \frac{t^2-1}{2} \quad t = 2t_1+1 \quad a = 4$ znak "−"

(21) $p_0^2 - 2tv_2p_0 - (1 - av_2^2) = 0 \quad p_0^2 - 2t\,\frac{t^2-1}{2}p_0 - \left[1 - 4\left(\frac{t^2-1}{2}\right)^2\right] = 0 \quad \sqrt{\triangle} = t(t^2-3)$

(22) $p_0^2 - 2tv_2p_0 + (1 + av_2^2) = 0 \quad p_0^2 - 2t\,\frac{t^2-1}{2}p_0 + \left[1 - 4\left(\frac{t^2-1}{2}\right)^2\right] = 0 \quad \sqrt{\triangle} = \sqrt{t^2(t^2-3)^2-8}$ odpada

$$p_0 = \frac{t(t^2-1)+t(t^2-3)}{2} = t(t^2-2) \qquad p_0 = \frac{t(t^2-1)-t(t^2-3)}{2} = t$$

Mamy więc:

$$v_2 = y = \frac{t^2-1}{2} \qquad p_0 = t$$

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot t - 4\,\frac{t^2-1}{2} = 2$$
$$p_2 = 2tp_1 - ap_0 = 2t\cdot 2 - 4t = 0$$
$$p_3 = 2tp_2 - ap_1 = 2t\cdot 0 - 4\cdot 2 = -8$$
$$p_4 = 2tp_3 - ap_2 = 2t(-8) - 4\cdot 0 = -16t$$
$$p_5 = 2tp_4 - ap_3 = 2t(-16t) - 4(-8) = -32(t^2-1)$$

przykłady

$D = 21 = 5^2 - 4 \quad t = 5 \quad a = 4$

$v_2 = y = 12 \quad 1\cdot 4^0 + 21\cdot 12^2 = 55^2$
$p_0 = 5 \quad 1\cdot 4^1 + 21\cdot 5^2 = 23^2$
$p_1 = 2 \quad 1\cdot 4^2 + 21\cdot 2^2 = 10^2$
$p_2 = 0 \quad 1\cdot 4^3 + 21\cdot 0^2 = 8^2$
$p_3 = -8 \quad 1\cdot 4^4 + 21(-8)^2 = 40^2$
$p_4 = -80 \quad 1\cdot 4^5 + 21(-80)^2 = 368^2$
$p_5 = -768 \quad 1\cdot 4^6 + 21(-768)^2 = 3520^2$

$D = 45 = 7^2 - 4 \quad t = 7 \quad a = 4$

$v_2 = y = 24 \quad 1\cdot 4^0 + 45\cdot 24^2 = 161^2$
$p_0 = 7 \quad 1\cdot 4^1 + 45\cdot 7^2 = 47^2$
$p_1 = 2 \quad 1\cdot 4^2 + 45\cdot 2^2 = 14^2$
$p_2 = 0 \quad 1\cdot 4^3 + 45\cdot 0^2 = 8^2$
$p_3 = -8 \quad 1\cdot 4^4 + 45(-8)^2 = 56^2$
$p_4 = -112 \quad 1\cdot 4^5 + 45(-112)^2 = 752^2$
$p_5 = -1536 \quad 1\cdot 4^6 + 45(-1536)^2 = 10304^2$

RGVIII

$D = (2t_1+1)^2 + 4 \quad v_2 = y = 4(2t_1+1)(t_1^2+t_1+1)(2t_1^2+2t_1+1) = t(t^2+3)\frac{t^2+1}{2} \quad t = 2t_1+1 \quad a = 4$ znak "+"

(15) $p_0^2 + 2tv_2p_0 - (1 + av_2^2) = 0 \quad p_0^2 + 2typ_0 - (1 + 4y^2) = 0 \quad \sqrt{\triangle} = (t^2+2)(t^4+4t^2+1)$

(16) $p_0^2 + 2tv_2p_0 + (1 - av_2^2) = 0 \quad p_0^2 + 2typ_0 + (1 - 4y^2) = 0 \quad \sqrt{\triangle} = \sqrt{(t^2+2)^2(t^4+4t^2+1)^2-8}$ odpada

$$p_0 = \frac{-2t\cdot t(t^2+3)\frac{t^2+1}{2} + (t^2+2)(t^4+3t^2+1)}{2} = (t^2+1)^2 + t^2$$

$$p_0 = \frac{-2t\cdot t(t^2+3)\frac{t^2+1}{2} - (t^2+2)(t^4+3t^2+1)}{2} = -(t^6+5t^4+6t^2+1)$$

Mamy więc: $v_2 = y = t(t^2+3)\frac{t^2+1}{2}$ $\qquad p_0 = (t^2+1)^2 + t^2$

(27)

$$p_1 = av_2 - 2tp_0 = 4t(t^2+3)\frac{t^2+1}{2} - 2t(t^4+3t^2+1) = 2t(t^2+2)$$

$$p_2 = ap_0 - 2tp_1 = 4(t^4+3t^2+1) - 2t\cdot 2t(t^2+2) = 4(t^2+1)$$

$$p_3 = ap_1 - 2tp_2 = 4\cdot 2t(t^2+2) - 2t\cdot 4(t^2+1) = 8t$$

$$p_4 = ap_2 - 2tp_3 = 4\cdot 4(t^2+1) - 2t\cdot 8t = 16$$

$$p_5 = ap_3 - 2tp_4 = 4\cdot 8t - 2t\cdot 16 = 0$$

przykłady

$D = 29 = 5^2+4 \quad t=5 \quad a=4$		$D = 53 = 7^2+4 \quad t=7 \quad a=4$	
$v_2 = y = 1820$	$+1\cdot 4^0 + 29\cdot 1820^2 = 9801^2$	$v_2 = y = 9100$	$+1\cdot 4^0 + 53\cdot 9100^2 = 66249^2$
$p_0 = 701$	$-1\cdot 4^1 + 29\cdot 701^2 = 3775^2$	$p_0 = 2549$	$-1\cdot 4^1 + 53\cdot 2549^2 = 18557^2$
$p_1 = 270$	$+1\cdot 4^2 + 29\cdot 270^2 = 1454^2$	$p_1 = 714$	$+1\cdot 4^2 + 53\cdot 714^2 = 5198^2$
$p_2 = 104$	$-1\cdot 4^3 + 29\cdot 104^2 = 560^2$	$p_2 = 200$	$-1\cdot 4^3 + 53\cdot 200^2 = 1456^2$
$p_3 = 40$	$+1\cdot 4^4 + 29\cdot 40^2 = 216^2$	$p_3 = 56$	$+1\cdot 4^4 + 53\cdot 56^2 = 408^2$
$p_4 = 16$	$-1\cdot 4^5 + 29\cdot 16^2 = 80^2$	$p_4 = 16$	$-1\cdot 4^5 + 53\cdot 16^2 = 112^2$
$p_5 = 0$	$+1\cdot 4^6 + 29\cdot 0^2 = 64^2$	$p_5 = 0$	$+1\cdot 4^6 + 53\cdot 0^2 = 64^2$

RGIX

$D = t^2 - \frac{t}{n} \qquad v_2 = y = 2n \qquad a = \frac{t}{n} \qquad$ znak "−"

(21) $p_0^2 - 2tv_2p_0 - (1 - av_2^2) = 0 \quad p_0^2 - 2t\cdot 2np_0 - \left[1 - \frac{t}{n}(2n)^2\right] = 0 \quad \sqrt{\triangle} = 2(2tn-1)$

(22) $p_0^2 - 2tv_2p_0 + (1 + av_2^2) = 0 \quad p_0^2 - 2t\cdot 2np_0 + \left[1 + \frac{t}{n}(2n)^2\right] = 0 \quad \sqrt{\triangle} = \sqrt{4(2tn-1)^2-8}$ odpada

$$p_0 = \frac{4tn + 2(2tn-1)}{2} = 4tn-1 \qquad p_0 = \frac{4tn - 2(2tn-1)}{2} = 1$$

Mamy więc: $v_2 = y = 2n \qquad p_0 = 1$

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot 1 - \frac{t}{n}\cdot 2n = 0$$

$$p_2 = 2tp_1 - ap_0 = 2t\cdot 0 - \frac{t}{n}\cdot 1 = -\frac{t}{n}$$

$$p_3 = 2tp_2 - ap_1 = 2t\left(-\frac{t}{n}\right) - \frac{t}{n}\cdot 0 = -\frac{2t^2}{n}$$

$$p_4 = 2tp_3 - ap_2 = 2t\left(-\frac{2t^2}{n}\right) - \frac{t}{n}\left(-\frac{t}{n}\right) = -\frac{t^2}{n^2}(4tn-1)$$

$$p_5 = 2tp_4 - ap_3 = 2t\left[-\frac{t^2}{n^2}(4tn-1)\right] - \frac{t}{n}\left(-\frac{2t^2}{n}\right) = -\frac{4t^3}{n^2}(2tn-1)$$

przykłady

$D = 20 = 5^2 - 5 = 5^2 - \frac{5}{1} \quad t=5 \quad a=5 \quad n=1$	$D = 1078 = 33^2 - 11 = 33^2 - \frac{33}{3} \quad t=33 \quad a=11 \quad n=3$
$v_2 = y = 2 \qquad 1\cdot 5^0 + 20\cdot 2^2 = 9^2$	$v_2 = y = 2 \quad 1\cdot 11^0 + 1078\cdot 6^2 = 197^2$

$p_0=1$ $\quad 1\cdot5^1+20\cdot1^2=5^2$ $\qquad p_0=1$ $\quad 1\cdot11^1+1078\cdot1^2=33^2$

$p_1=0$ $\quad 1\cdot5^2+20\cdot0^2=5^2$ $\qquad p_1=0$ $\quad 1\cdot11^2+1078\cdot0^2=11^2$

$p_2=-5$ $\quad 1\cdot5^3+20(-5)^2=25^2$ $\qquad p_2=-11$ $\quad 1\cdot11^3+1078(-11)^2=363^2$

$p_3=-50$ $\quad 1\cdot5^4+20(-50)^2=225^2$ $\qquad p_3=-726$ $\quad 1\cdot11^4+1078(-726)^2=23837^2$

$p_4=-475$ $\quad 1\cdot5^5+20(-475)^2=2125^2$ $\qquad p_4=-47795$ $\quad 1\cdot11^5+1078(-47795)^2=1569249^2$

$p_5=-4500$ $\quad 1\cdot5^6+20(-4500)^2=20125^2$ $\qquad p_5=-3146484$ $\quad 1\cdot11^6+1078(-3146484)^2=103308227^2$

RGIX

$$D=t^2+\frac{t}{n} \qquad v_2=y=2n \qquad a=\frac{t}{n} \qquad \text{znak "+"}$$

(15) $p_0^2+2tv_2p_0-(1+av_2^2)=0 \qquad p_0^2+2t\cdot2np_0-\left[1+\frac{t}{n}(2n)^2\right]=0 \qquad \sqrt{\triangle}=2(2tn+1)$

(16) $p_0^2+2tv_2p_0+(1-av_2^2)=0 \qquad p_0^2+2t\cdot2np_0+\left[1-\frac{t}{n}(2n)^2\right]=0 \qquad \sqrt{\triangle}=\sqrt{4(2tn+1)^2-8}$ odpada

$$p_0=\frac{-4tn+2(2tn+1)}{2}=1 \qquad p_0=\frac{-4tn-2(2tn+1)}{2}=-(4tn+1)$$

Mamy więc:

$$v_2=y=2n \qquad p_0=1$$

(27)

$$p_1=av_2-2tp_0=\frac{t}{n}2n-2t\cdot1=0$$

$$p_2=ap_0-2tp_1=\frac{t}{n}\cdot1-2t\cdot0=\frac{t}{n}$$

$$p_3=ap_1-2tp_2=\frac{t}{n}\cdot0-2t\cdot\frac{t}{n}=-\frac{2t^2}{n}$$

$$p_4=ap_2-2tp_3=\frac{t}{n}\cdot\frac{t}{n}-2t\left(-\frac{2t^2}{n}\right)=\frac{t^2}{n^2}(4tn+1)$$

$$p_5=ap_3-2tp_4=\frac{t}{n}\left(-\frac{2t^2}{n}\right)-2t\left[-\frac{t^2}{n^2}(4tn+1)\right]=-\frac{4t^3}{n^2}(2tn+1)$$

przykłady

$D=30=5^2+5=5^2+\frac{5}{1}$ $\quad t=5 \quad a=5 \quad n=1$ $\qquad D=203=14^2+7=14^2+\frac{14}{2}$ $\quad t=14 \quad a=7 \quad n=2$

$v_2=y=2$ $\quad +1\cdot5^0+30\cdot2^2=11^2$ $\qquad v_2=y=4$ $\quad +1\cdot7^0+203\cdot4^2=57^2$

$p_0=1$ $\quad -1\cdot5^1+30\cdot1^2=5^2$ $\qquad p_0=1$ $\quad -1\cdot7^1+203\cdot1^2=14^2$

$p_1=0$ $\quad +1\cdot5^2+30\cdot0^2=5^2$ $\qquad p_1=0$ $\quad +1\cdot7^2+203\cdot0^2=7^2$

$p_2=5$ $\quad -1\cdot5^3+30\cdot5^2=25^2$ $\qquad p_2=7$ $\quad -1\cdot7^3+203\cdot7^2=98^2$

$p_3=-50$ $\quad +1\cdot5^4+30(-50)^2=275^2$ $\qquad p_3=-196$ $\quad +1\cdot7^4+203(-196)^2=2793^2$

$p_4=525$ $\quad -1\cdot5^5+30\cdot525^2=2875^2$ $\qquad p_4=5537$ $\quad -1\cdot7^5+203\cdot5537^2=78890^2$

$p_5=-5500$ $\quad +1\cdot5^6+30(-5500)^2=30125^2$ $\qquad p_5=-156408$ $\quad +1\cdot7^6+203(-156408)^2=2228471^2$

RGX

$$D=t^2-\frac{2t}{n} \qquad v_2=y=n \qquad a=\frac{2t}{n} \qquad \text{znak "–"}$$

(21) $p_0^2-2tv_2p_0-(1-av_2^2)=0 \qquad p_0^2-2tnp_0-\left(1-\frac{2t}{n}n^2\right)=0 \qquad \sqrt{\triangle}=2(tn-1)$

(22) $p_0^2-2tv_2p_0+(1+av_2^2)=0 \qquad p_0^2-2tnp_0+\left(1+\frac{2t}{n}n^2\right)=0 \qquad \sqrt{\triangle}=\sqrt{2(tn-1)^2-8}$ odpada

$$p_0=\frac{2tn+2(tn-1)}{2}=2tn-1 \qquad p_0=\frac{2tn-2(tn-1)}{2}=1$$

Mamy więc:

$$v_2 = y = n \qquad p_0 = 1$$

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot 1 - \frac{2t}{n}n = 0$$

$$p_2 = 2tp_1 - ap_0 = 2t\cdot 0 - \frac{2t}{n}\cdot 1 = -\frac{2t}{n}$$

$$p_3 = 2tp_2 - ap_1 = 2t\left(-\frac{2t}{n}\right) - \frac{2t}{n}\cdot 0 = -\frac{4t^2}{n}$$

$$p_4 = 2tp_3 - ap_2 = 2t\left(-\frac{4t^2}{n}\right) - \frac{2t}{n}\left(-\frac{2t}{n}\right) = -\frac{4t^2}{n^2}\left(2tn-1\right)$$

$$p_5 = 2tp_4 - ap_3 = 2t\left[-\frac{4t^2}{n^2}\left(2tn-1\right)\right] - \frac{2t}{n}\left(-\frac{4t^2}{n}\right) = -\frac{16t^3}{n^2}\left(tn-1\right)$$

przykłady

$D = 435 = 21^2 - 6 = 21^2 - \frac{2\cdot 21}{7}$ $t = 21$ $a = 6$ $n = 7$

$v_2 = y = 7$	$1\cdot 6^0 + 435\cdot 7^2 = 146^2$
$p_0 = 1$	$1\cdot 6^1 + 435\cdot 1^2 = 21^2$
$p_1 = 0$	$1\cdot 6^2 + 435\cdot 0^2 = 6^2$
$p_2 = -6$	$1\cdot 6^3 + 435(-6)^2 = 126^2$
$p_3 = -252$	$1\cdot 6^4 + 435(-252)^2 = 5256^2$
$p_4 = -10548$	$1\cdot 6^5 + 435(-10548)^2 = 219996^2$
$p_5 = -441504$	$1\cdot 6^6 + 435(-441504)^2 = 9208296^2$

$D = 3003 = 55^2 - 22 = 55^2 - \frac{2\cdot 55}{5}$ $t = 55$ $a = 22$ $n = 5$

$v_2 = y = 5$	$1\cdot 22^0 + 3003\cdot 5^2 = 274^2$
$p_0 = 1$	$1\cdot 22^1 + 3003\cdot 1^2 = 55^2$
$p_1 = 0$	$1\cdot 22^2 + 3003\cdot 0^2 = 22^2$
$p_2 = -22$	$1\cdot 22^3 + 3003(-22)^2 = 1210^2$
$p_3 = -2420$	$1\cdot 22^4 + 3003(-2420)^2 = 132616^2$
$p_4 = -265716$	$1\cdot 22^5 + 3003(-265716)^2 = 14561140^2$
$p_5 = -29175520$	$1\cdot 22^6 + 3003(-29175520)^2 = 1598807848^2$

RGX

$$D = t^2 + \frac{2t}{n} \qquad v_2 = y = n \qquad a = \frac{2t}{n} \qquad \text{znak "+"}$$

(15) $p_0^2 + 2tv_2p_0 - (1 + av_2^2) = 0$ $\quad p_0^2 + 2tnp_0 - \left(1 + \frac{2t}{n}n^2\right) = 0$ $\quad \sqrt{\triangle} = 2(tn+1)$

(16) $p_0^2 + 2tv_2p_0 + (1 - av_2^2) = 0$ $\quad p_0^2 + 2tnp_0 + \left(1 - \frac{2t}{n}n^2\right) = 0$ $\quad \sqrt{\triangle} = \sqrt{4(tn+1)^2 - 8}$ odpada

$$p_0 = \frac{-2tn + 2(tn+1)}{2} = 1 \qquad p_0 = \frac{-2tn - 2(tn+1)}{2} = -(2tn+1)$$

Mamy więc:

$$v_2 = y = n \qquad p_0 = 1$$

(27)

$$p_1 = av_2 - 2tp_0 = \frac{2t}{n}n - 2t\cdot 1 = 0$$

$$p_2 = ap_0 - 2tp_1 = \frac{2t}{n}\cdot 1 - 2t\cdot 0 = \frac{2t}{n}$$

$$p_3 = ap_1 - 2tp_2 = \frac{2t}{n}\cdot 0 - 2t\cdot\frac{2t}{n} = -\frac{4t^2}{n}$$

$$p_4 = ap_2 - 2tp_3 = \frac{2t}{n}\cdot\frac{2t}{n} - 2t\left(-\frac{4t^2}{n}\right) = \frac{4t^2}{n^2}\left(2tn+1\right)$$

$$p_5 = ap_3 - 2tp_4 = \frac{2t}{n}\left(-\frac{4t^2}{n}\right) - 2t\left[\frac{4t^2}{n^2}\left(2tn+1\right)\right] = -\frac{16t^3}{n^2}\left(tn+1\right)$$

przykłady

$D = 447 = 21^2 + 6 = 21^2 + \frac{2\cdot 21}{7}$ $t = 21$ $a = 6$ $n = 7$

$v_2 = y = 7$	$+1\cdot 6^0 + 447\cdot 7^2 = 148^2$
$p_0 = 1$	$-1\cdot 6^1 + 447\cdot 1^2 = 21^2$

$D = 3047 = 55^2 + 22 = 55^2 + \frac{2\cdot 55}{5}$ $t = 55$ $a = 22$ $n = 5$

$v_2 = y = 5$	$+1\cdot 22^0 + 3047\cdot 5^2 = 276^2$
$p_0 = 1$	$-1\cdot 22^1 + 3047\cdot 1^2 = 55^2$

$p_1 = 0 \quad + 1\cdot 6^2 + 447\cdot 0^2 = 6^2$

$p_2 = 6 \quad - 1\cdot 6^3 + 447\cdot 6^2 = 126^2$

$p_3 = -252 \quad + 1\cdot 6^4 + 447(-252)^2 = 5328^2$

$p_4 = 10620 \quad - 1\cdot 6^5 + 447\cdot 10620^2 = 224532^2$

$p_5 = -447552 \quad + 1\cdot 6^6 + 447(-447552)^2 = 9462312^2$

$p_1 = 0 \quad + 1\cdot 22^2 + 3047\cdot 0^2 = 22^2$

$p_2 = 22 \quad - 1\cdot 22^3 + 3047\cdot 22^2 = 1210^2$

$p_3 = -2420 \quad + 1\cdot 22^4 + 3047(-2420)^2 = 133584^2$

$p_4 = 266684 \quad - 1\cdot 22^5 + 3047\cdot 266684^2 = 14720860^2$

$p_5 = -29388480 \quad + 1\cdot 22^6 + 3047(-29388480)^2 = 1622233448^2$

RGXI

$D = t^2 - \frac{4t}{n}$ $\quad v_2 = y = \frac{tn-2}{2}n$ $\quad tn$ — parzyste $\quad a = \frac{4t}{n}$ $\quad$ znak "–"

(19) $p_0^2 - 2tv_2p_0 - (1 - av_2^2) = 0 \quad p_0^2 - 2typ_0 - \left(1 - \frac{4t}{n}y^2\right) = 0 \quad \sqrt{\triangle} = (tn-2)^2 - 2$

(24) $p_0^2 - 2tv_2p_0 + (1 + av_2^2) = 0 \quad p_0^2 - 2typ_0 + \left(1 + \frac{4t}{n}y^2\right) = 0 \quad \sqrt{\triangle} = \sqrt{\left[(tn-2)^2-2\right]^2 - 8}$ odpada

$$p_0 = \frac{2t\frac{tn-2}{2}n + (tn-2)^2 - 2}{2} = (tn-2)^2 - (tn+2) \qquad p_0 = \frac{2t\frac{tn-2}{2}n - \left[(tn-2)^2-2\right]}{2} = tn-1$$

Mamy więc:

$$v_2 = y = \frac{tn-2}{2}n \qquad p_0 = tn-1$$

(28)

$$p_1 = 2tp_0 - av_2 = 2t(tn-1) - \frac{4t}{n}\cdot\frac{tn-2}{2}n = 2t$$

$$p_2 = 2tp_1 - ap_0 = 2t\cdot 2t - \frac{4t}{n}(tn-1) = \frac{4t}{n}$$

$$p_3 = 2tp_2 - ap_1 = 2t\frac{4t}{n} - \frac{4t}{n}2t = 0$$

$$p_4 = 2tp_3 - ap_2 = 2t\cdot 0 - \frac{4t}{n}\cdot\frac{4t}{n} = -\left(\frac{4t}{n}\right)^2$$

$$p_5 = 2tp_4 - ap_3 = 2t\left(-\frac{16t^2}{n^2}\right) - \frac{4t}{n}\cdot 0 = -\frac{32t^3}{n^2}$$

przykłady

$D = 4860 = 70^2 - \frac{4\cdot 70}{7} \quad t = 70 \quad a = 40 \quad n = 7$

$v_2 = y = 1708 \quad 1\cdot 40^0 + 4860\cdot 1708^2 = 119071^2$

$p_0 = 489 \quad 1\cdot 40^1 + 4860\cdot 489^2 = 34090^2$

$p_1 = 140 \quad 1\cdot 40^2 + 4860\cdot 140^2 = 9760^2$

$p_2 = 40 \quad 1\cdot 40^3 + 4860\cdot 40^2 = 2800^2$

$p_3 = 0 \quad 1\cdot 40^4 + 4860\cdot 0^2 = 1600^2$

$p_4 = -1600 \quad 1\cdot 40^5 + 4860(-1600)^2 = 112000^2$

$p_5 = -224000 \quad 1\cdot 40^6 + 4860(-224000)^2 = 15616000^2$

$D = 384 = 20^2 - \frac{4\cdot 20}{5} \quad t = 20 \quad a = 16 \quad n = 5$

$v_2 = y = 245 \quad 1\cdot 16^0 + 384\cdot 245^2 = 4801^2$

$p_0 = 99 \quad 1\cdot 16^1 + 384\cdot 99^2 = 1940^2$

$p_1 = 40 \quad 1\cdot 16^2 + 384\cdot 40^2 = 784^2$

$p_2 = 16 \quad 1\cdot 16^3 + 384\cdot 16^2 = 320^2$

$p_3 = 0 \quad 1\cdot 16^4 + 384\cdot 0^2 = 256^2$

$p_4 = -256 \quad 1\cdot 16^5 + 384(-256)^2 = 5120^2$

$p_5 = -10240 \quad 1\cdot 16^6 + 384(-10240)^2 = 200704^2$

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$D = t^2 - \frac{4t}{n}$ $\quad v_2 = y = \frac{(tn-2)^2-1}{2}n$ $\quad tn$ — nieparzyste $\quad a = \frac{4t}{n}$ $\quad$ znak "–"

(19) $p_0^2 - 2tv_2p_0 - (1 - av_2^2) = 0 \quad p_0^2 - 2typ_0 - \left(1 - \frac{4t}{n}y^2\right) = 0 \quad \sqrt{\triangle} = (tn-2)(t^2n^2 - 4tn + 1)$

(24) $p_0^2 - 2tv_2p_0 + (1 + av_2^2) = 0 \quad p_0^2 - 2typ_0 + \left(1 + \frac{4t}{n}y^2\right) = 0 \quad \sqrt{\triangle} = \sqrt{\left[(tn-2)(t^2n^2 - 4tn + 1)\right] - 8}$ odpada

$$p_0 = \frac{2t\frac{(tn-2)^2-1}{2}n + (tn-2)(t^2n^2 - 4tn + 1)}{2} = t^3n^3 - 5t^2n^2 + 6tn - 1$$

$$p_0 = \frac{2t\frac{(tn-2)^2-1}{2}n - (tn-2)(t^2n^2-4tn+1)}{2} = (tn-1)^2 - tn$$

Mamy więc:

$$v_2 = y = \frac{(tn-2)^2-1}{2}n \qquad p_0 = (tn-1)^2 - tn$$

(28)

$$p_1 = 2tp_0 - av_2 = 2t\left[(tn-1)^2 - tn\right] - \frac{4t}{n}\cdot\frac{(tn-2)^2-1}{2}n = 2t(tn-2)$$

$$p_2 = 2tp_1 - ap_0 = 2t\cdot 2t(tn-2) - \frac{4t}{n}\left[(tn-1)^2 - tn\right] = \frac{4t}{n}\left(tn-1\right)$$

$$p_3 = 2tp_2 - ap_1 = 2t\,\frac{4t}{n}\left(tn-1\right) - \frac{4t}{n}2t(tn-2) = \frac{8t^2}{n}$$

$$p_4 = 2tp_3 - ap_2 = 2t\frac{8t^2}{n} - \frac{4t}{n}\cdot\frac{4t}{n}\left(tn-1\right) = \left(\frac{4t}{n}\right)^2$$

$$p_5 = 2tp_4 - ap_3 = 2t\,\frac{16t^2}{n^2} - \frac{4t}{n}\cdot\frac{8t^2}{n} = 0$$

przykłady

$D = 429 = 21^2 - \frac{4\cdot 21}{7}$ $\quad t=21 \quad a=12 \quad n=7$		$D = 10965 = 105^2 - \frac{4\cdot 105}{7}$ $\quad t=105 \quad a=60 \quad n=7$	
$v_2 = y = 73584$	$1\cdot 12^0 + 429\cdot 73584^2 = 1524095^2$	$v_2 = y = 1880508$	$1\cdot 60^0 + 10965\cdot 1880508^2 = 196915319^2$
$p_0 = 21169$	$1\cdot 12^1 + 429\cdot 21169^2 = 438459^2$	$p_0 = 538021$	$1\cdot 60^1 + 10965\cdot 538021^2 = 56338275^2$
$p_1 = 6090$	$1\cdot 12^2 + 429\cdot 6090^2 = 126138^2$	$p_1 = 153930$	$1\cdot 60^2 + 10965\cdot 153930^2 = 16118610^2$
$p_2 = 1752$	$1\cdot 12^3 + 429\cdot 1752^2 = 36288^2$	$p_2 = 44040$	$1\cdot 60^3 + 10965\cdot 44040^2 = 4611600^2$
$p_3 = 504$	$1\cdot 12^4 + 429\cdot 504^2 = 10440^2$	$p_3 = 12600$	$1\cdot 60^4 + 10965\cdot 12600^2 = 1319400^2$
$p_4 = 144$	$1\cdot 12^5 + 429\cdot 144^2 = 3024^2$	$p_4 = 3600$	$1\cdot 60^5 + 10965\cdot 3600^2 = 378000^2$
$p_5 = 0$	$1\cdot 12^6 + 429\cdot 0^2 = 1728^2$	$p_5 = 0$	$1\cdot 60^6 + 10965\cdot 0^2 = 216000^2$

RGXI

$D = t^2 + \frac{4t}{n} \qquad v_2 = y = \frac{tn+2}{2}n \qquad tn$ — parzyste $\qquad a = \frac{4t}{n} \qquad$ znak "+"

(13) $\quad p_0^2 + 2tv_2p_0 - (1 + av_2^2) = 0 \qquad p_0^2 + 2typ_0 - \left(1 + \frac{t}{n}y^2\right) = 0 \qquad \sqrt{\triangle} = (tn+2)^2 - 2$

(18) $\quad p_0^2 + 2tv_2p_0 + (1 - av_2^2) = 0 \qquad p_0^2 + 2typ_0 + \left(1 - \frac{t}{n}y^2\right) = 0 \qquad \sqrt{\triangle} = \sqrt{\left[(tn+2)^2-2\right]^2 - 8}$ odpada

$$p_0 = \frac{-2t\frac{tn+2}{2}n + (tn+2)^2 - 2}{2} = tn+1 \qquad p_0 = \frac{-2t\frac{tn+2}{2}n - \left[(tn+2)^2-2\right]}{2} = -\left[(tn+1)^2 + tn\right]$$

Mamy więc:

$$v_2 = y = \frac{tn+2}{2}n \qquad p_0 = tn+1$$

(27)

$$p_1 = av_2 - 2tp_0 = \frac{4t}{n}\cdot\frac{tn+2}{2}n - 2t(tn+1) = 2t$$

$$p_2 = ap_0 - 2tp_1 = \frac{4t}{n}(tn+1) - 2t\cdot 2t = \frac{4t}{n}$$

$$p_3 = ap_1 - 2tp_2 = \frac{4t}{n}\cdot 2t - 2t\frac{4t}{n} = 0$$

$$p_4 = ap_2 - 2tp_3 = \frac{4t}{n}\cdot\frac{4t}{n} - 2t\cdot 0 = \left(\frac{4t}{n}\right)^2$$

$$p_5 = ap_3 - 2tp_4 = \frac{4t}{n}\cdot 0 - 2t\,\frac{16t^2}{n^2} = -\frac{32t^3}{n^2}$$

przykłady

$D = 160 = 12^2 + \frac{4\cdot 12}{3}$ $t=12$ $a=16$ $n=3$

$v_2 = y = 57$ $+1\cdot 16^0 + 160\cdot 57^2 = 721^2$
$p_0 = 37$ $-1\cdot 16^1 + 160\cdot 37^2 = 468^2$
$p_1 = 24$ $+1\cdot 16^2 + 160\cdot 24^2 = 304^2$
$p_2 = 16$ $-1\cdot 16^3 + 160\cdot 16^2 = 192^2$
$p_3 = 0$ $+1\cdot 16^4 + 160\cdot 0^2 = 256^2$
$p_4 = 256$ $-1\cdot 16^5 + 160\cdot 256^2 = 3072^2$
$p_5 = -6144$ $+1\cdot 16^6 + 160(-6144)^2 = 77824^2$

$D = 416 = 20^2 + \frac{4\cdot 20}{5}$ $t=20$ $a=16$ $n=5$

$v_2 = y = 255$ $+1\cdot 16^0 + 416\cdot 255^2 = 5201^2$
$p_0 = 101$ $-1\cdot 16^1 + 416\cdot 101^2 = 2060^2$
$p_1 = 40$ $+1\cdot 16^2 + 416\cdot 40^2 = 816^2$
$p_2 = 16$ $-1\cdot 16^3 + 416\cdot 16^2 = 320^2$
$p_3 = 0$ $+1\cdot 16^4 + 416\cdot 0^2 = 256^2$
$p_4 = 256$ $-1\cdot 16^5 + 416\cdot 256^2 = 5120^2$
$p_5 = -10240$ $+1\cdot 16^6 + 416(-10240)^2 = 208896^2$

— — — — — — — — — — — —

$D = t^2 + \frac{4t}{n}$ $v_2 = y = \frac{(tn+2)^2-1}{2}\,n$ tn — nieparzyste $a = \frac{4t}{n}$ znak "+"

(13) $p_0^2 + 2tv_2p_0 - (1 + av_2^2) = 0$ $p_0^2 + 2typ_0 - \left(1 + \frac{t}{n}y^2\right) = 0$ $\sqrt{\triangle} = (tn+2)(t^2n^2 + 4tn + 1)$

(18) $p_0^2 + 2tv_2p_0 + (1 - av_2^2) = 0$ $p_0^2 + 2typ_0 + \left(1 - \frac{t}{n}y^2\right) = 0$ $\sqrt{\triangle} = \sqrt{(tn+2)^2(t^2n^2 + 4tn + 1)^2 - 8}$ odpada

$$p_0 = \frac{-2t\frac{(tn+2)^2-1}{2}\,n + (tn+2)(t^2n^2+4tn+1)}{2} = (tn+1)^2 + tn$$

$$p_0 = \frac{-2t\frac{(tn+2)^2-1}{2}\,n - (tn+2)(t^2n^2+4tn+1)}{2} = -(t^3n^3 + 5t^2n^2 + 6tn + 1)$$

Mamy więc: $v_2 = y = \frac{(tn+2)^2-1}{2}\,n$ $p_0 = (tn+1)^2 + tn$

(27)

$$p_1 = av_2 - 2tp_0 = \frac{4t}{n}\cdot\frac{(tn+2)^2-1}{2}\,n - 2t\left[(tn+1)^2 + tn\right] = 2t(tn+2)$$

$$p_2 = ap_0 - 2tp_1 = \frac{4t}{n}\left[(tn+1)^2 + tn\right] - 2t\cdot 2t(tn+2) = \frac{4t}{n}(tn+1)$$

$$p_3 = ap_1 - 2tp_2 = \frac{4t}{n}\,2t(tn+2) - 2t\cdot\frac{4t}{n}(tn+1) = \frac{8t^2}{n}$$

$$p_4 = ap_2 - 2tp_3 = \frac{4t}{n}\cdot\frac{4t}{n}(tn+1) - 2t\cdot\frac{8t^2}{n} = \left(\frac{4t}{n}\right)^2$$

$$p_5 = ap_3 - 2tp_4 = \frac{4t}{n}\cdot\frac{8t^2}{n} - 2t\,\frac{16t^2}{n^2} = 0$$

przykłady

$D = 245 = 15^2 + \frac{4\cdot 15}{3}$ $t=15$ $a=20$ $n=3$

$v_2 = y = 3312$ $+1\cdot 20^0 + 245\cdot 3312^2 = 51841^2$
$p_0 = 2161$ $-1\cdot 20^1 + 245\cdot 2161^2 = 33825^2$
$p_1 = 1410$ $+1\cdot 20^2 + 245\cdot 1410^2 = 22070^2$
$p_2 = 920$ $-1\cdot 20^3 + 245\cdot 920^2 = 14400^2$
$p_3 = 600$ $+1\cdot 20^4 + 245\cdot 600^2 = 9400^2$
$p_4 = 400$ $-1\cdot 20^5 + 245\cdot 400^2 = 6000^2$
$p_5 = 0$ $+1\cdot 20^6 + 245\cdot 0^2 = 8000^2$

$D = 453 = 21^2 + \frac{4\cdot 21}{7}$ $t=21$ $a=12$ $n=7$

$v_2 = y = 77700$ $+1\cdot 12^0 + 453\cdot 77700^2 = 1653751^2$
$p_0 = 22051$ $-1\cdot 12^1 + 453\cdot 22051^2 = 469329^2$
$p_1 = 6258$ $+1\cdot 12^2 + 453\cdot 6258^2 = 133194^2$
$p_2 = 1776$ $-1\cdot 12^3 + 453\cdot 1776^2 = 37800^2$
$p_3 = 504$ $+1\cdot 12^4 + 453\cdot 504^2 = 10728^2$
$p_4 = 144$ $-1\cdot 12^5 + 453\cdot 144^2 = 3024^2$
$p_5 = 0$ $+1\cdot 12^6 + 453\cdot 0^2 = 1728^2$

Na następnej stronie przedstawiono wyniki z tego podrozdziału.

Rozwiązania główne RGI–XI

D	$y = v_2$ $(v_1 = 1)$	p_0	p_1	p_2	p_3	p_4	p_5	gr.
t^2-1	1	$+$ 0	$+$ -1	$+$ $-2t$	$+$ $-(4t^2-1)$	$+$ $-4t(2t^2-1)$	$+$ $-4t^2(4t^2-3)-1$	I
t^2+1	$2t$	$-$ 1	$+$ 0	$-$ 1	$+$ $-2t$	$-$ $4t^2+1$	$+$ $-4t(2t^2+1)$	II
t^2-2	t	$+$ 1	$+$ 0	$+$ -2	$+$ $-4t$	$+$ $-4(2t^2-1)$	$+$ $-16t(t^2-1)$	III
t^2+2	t	$-$ 1	$+$ 0	$-$ 2	$+$ $-4t$	$-$ $4(2t^2+1)$	$+$ $-16t(t^2+1)$	IV
$(2t_1)^2-4$	t_1	$+$ 1	$+$ 0	$+$ -4	$+$ $-8t$	$+$ $-16(t^2-1)$	$+$ $-32t(t^2-2)$	V
$(2t_1)^2+4$	t_1	$-$ 1	$+$ 0	$-$ 4	$+$ $-8t$	$-$ $16(t^2+1)$	$+$ $-32t(t^2+2)$	VI
$(2t_1+1)^2-4$	$\frac{(2t_1+1)^2-1}{2}$ $2t_1(t_1+1)$	$+$ t	$+$ 2	$+$ 0	$+$ -8	$+$ $-16t$	$+$ $-32(t^2-1)$	VII
$(2t_1+1)^2+4$	$4(2t_1+1)(t_1^2+t_1+1)$ $\times(2t_1^2+2t_1+1)$	$-$ $(t^2+1)^2+t^2$	$+$ $2t(t^2+2)$	$-$ $4(t^2+1)$	$+$ $8t$	$-$ 16	$+$ 0	VIII
$t^2-\frac{t}{n}$	$2n$ $-$	$+$ 1	$+$ 0	$+$ $-\frac{t}{n}$	$+$ $-\frac{2t^2}{n}$	$+$ $-\left(\frac{t}{n}\right)^2(4tn-1)$	$+$ $-\frac{4t^3}{n^2}(2tn-1)$	IX
$t^2+\frac{t}{n}$	$2n$ $+$	$-$ 1	$+$ 0	$-$ $\frac{t}{n}$	$+$ $-\frac{2t^2}{n}$	$-$ $\left(\frac{t}{n}\right)^2(4tn+1)$	$+$ $-\frac{4t^3}{n^2}(2tn+1)$	
$t^2-\frac{2t}{n}$	n $-$	$+$ 1	$+$ 0	$+$ $-\frac{2t}{n}$	$+$ $-\frac{4t^2}{n}$	$+$ $-\left(\frac{2t}{n}\right)^2(2tn-1)$	$+$ $-\frac{16t^3}{n^2}(tn-1)$	X
$t^2+\frac{2t}{n}$	n $+$	$-$ 1	$+$ 0	$-$ $\frac{2t}{n}$	$+$ $-\frac{4t^2}{n}$	$-$ $\left(\frac{2t}{n}\right)^2(2tn+1)$	$+$ $-\frac{16t^3}{n^2}(tn+1)$	
$t^2-\frac{4t}{n}$	tn — parzyste $y=\frac{tn-2}{2}n$ $-$	$+$ $tn-1$	$+$ $2t$	$+$ $\frac{4t}{n}$	$+$ 0	$+$ $-\left(\frac{4t}{n}\right)^2$	$+$ $-\frac{32t^3}{n^2}$	XI
	tn — nieparzyste $y=\frac{(tn-2)^2-1}{2}n$ $-$	$-$ $(tn-1)^2-tn$	$+$ $2t(tn-2)$	$-$ $\frac{4t}{n}(tn-1)$	$+$ $\frac{8t^2}{n}$	$-$ $\left(\frac{4t}{n}\right)^2$	$+$ 0	
$t^2+\frac{4t}{n}$	tn — parzyste $y=\frac{tn+2}{2}n$ $+$	$+$ $tn+1$	$+$ $2t$	$+$ $\frac{4t}{n}$	$+$ 0	$+$ $\left(\frac{4t}{n}\right)^2$	$+$ $-\frac{32t^3}{n^2}$	XI
	tn — nieparzyste $y=\frac{(tn+2)^2-1}{2}n$ $+$	$-$ $(tn+1)^2+tn$	$+$ $2t(tn+2)$	$-$ $\frac{4t}{n}(tn+1)$	$+$ $\frac{8t^2}{n}$	$-$ $\left(\frac{4t}{n}\right)^2$	$+$ 0	

Podrozdział III

v_2 w grupie "±4−1"

W niektórych przypadkach RG v_2 może być częścią y (większą niż 1 i mniejszą niż y). Omówimy po kolei te przypadki.

RGII

$$D = t^2+1 \qquad y = v_1 v_2 = 2t \qquad \alpha = 1 \qquad \text{znak "+"}$$

Omówiliśmy ten przypadek dla $v_2 = 1$ i $v_2 = 2t$. Możemy jeszcze rozpatrzyć przypadki kiedy

$$v_2 = t$$
$$v_2 = 2$$

Nie wnoszą one nic nowego do metody rozwiązywania RP dla $D = t^2+1$, a jedynie rozkładają ten proces na dwa etapy. Mnożąc D przez $v_1^2 = 2^2$ otrzymujemy nowe D postaci t^2+4. Mnożąc D przez $v_1^2 = t^2$ otrzymujemy nowe D postaci t^2+t. Oba przypadki zostały omówione w poprzednich podrozdziałach.

RGVII

$$D = (2t_1+1)^2 - 4 \qquad v_2 = 2t_1(t_1+1) \qquad t = 2t_1+1 \qquad \alpha = 4 \qquad \text{znak "−"}$$

W tym przypadku mnożenie przez składniki $v_2 = 2t_1(t_1+1)$ tzn. $2t_1$ lub t_1+1 nie daje D takich, które podlegają pod RG.

RGVIII

$$D = (2t_1+1)^2 + 4 \qquad v_2 = 4(2t_1+1)(t_1^2+t_1+1)(2t_1^2+2t_1+1) \qquad t = 2t_1+1 \qquad \alpha = 4 \qquad \text{znak "+"}$$

$$v_2 = 2t_1+1 = t$$
$$v_2 = 2t_1^2+2t_1+1 = \frac{t^2+1}{2}$$
$$v_2 = 2(2t_1^2+2t_1+1) = t^2+1$$

Należą tutaj liczby z grupy "−4,−1,+4". W podrozdziale I i II rozpatrzyliśmy przypadki, kiedy odpowiednio:

$$v_2 = 1$$
$$v_2 = 4(2t_1+1)(t_1^2+t_1+1)(2t_1^2+2t_1+1)$$

Pozostają do rozpatrzenia przypadki kiedy:

$v_2 = 2t_1+1 = t$	przekształca D w liczbę postaci	$D = (2t_1+1)^2 - 4$	grupa "+4"
$v_2 = 2t_1^2+2t_1+1 = \frac{t^2+1}{2}$	przekształca D w liczbę postaci	$D = t^2+1$	grupa "−1"
$v_2 = 2(2t_1^2+2t_1+1) = t^2+1$	przekształca D w liczbę postaci	$D = (2t_1)^2 + 4$	grupa "−4"

Odpowiednie równania to:

(13) $p_0^2 + 2tv_2p_0 - (4+\alpha v_2^2) = 0 \qquad p_0^2 + 2t\cdot tp_0 - (4+4t^2) = 0 \qquad \sqrt{\triangle} = 2(t^2+2)$

(16) $p_0^2 + 2tv_2p_0 + (1-\alpha v_2^2) = 0 \qquad p_0^2 + 2t\,\frac{t^2+1}{2}\,p_0 + \left[1 - 4\left(\frac{t^2+1}{2}\right)^2\right] = 0 \qquad \sqrt{\triangle} = t(t^2+3)$

(18) $p_0^2 + 2tv_2p_0 + (4-\alpha v_2^2) = 0 \qquad p_0^2 + 2t(t^2+1)p_0 + \left[4 - 4(t^2+1)^2\right] = 0 \qquad \sqrt{\triangle} = 2t(t^2+3)$

(13) $p_0 = \frac{-2t\cdot t + 2(t^2+2)}{2} = 2$ $\qquad p_0 = \frac{-2t\cdot t - 2(t^2+2)}{2} = -(2t^2+2)$ odpada

(16) $p_0 = \frac{-2t\,\frac{t^2+1}{2} + t(t^2+3)}{2} = t$ $\qquad p_0 = \frac{-2t\,\frac{t^2+1}{2} - t(t^2+3)}{2} = -t(t^2+2)$ odpada

(18) $p_0 = \frac{-2t(t^2+1) + 2t(t^2+3)}{2} = 2t$ $\qquad p_0 = \frac{-2t(t^2+1) - 2t(t^2+3)}{2} = -2t(t^2+2)$ odpada

Mamy więc:

$v_2 = t$	$p_0 = 2$
$v_2 = \frac{t^2+1}{2}$	$p_0 = t$
$v_2 = t^2+1$	$p_0 = 2t$

$v_2 = t \qquad p_0 = 2$

(27)

$p_1 = av_2 - 2tp_0 = 4t - 2t\cdot 2 = 0$

$p_2 = ap_0 - 2tp_1 = 4\cdot 2 - 2t\cdot 0 = 8$

$p_3 = ap_1 - 2tp_2 = 4\cdot 0 - 2t\cdot 8 = -16t$

$p_4 = ap_2 - 2tp_3 = 4\cdot 8 - 2t(-16t) = 32(t^2+1)$

$p_5 = ap_3 - 2tp_4 = 4(-16t) - 2t\cdot 32(t^2+1) = -64t(t^2+2)$

przykłady

$D = 29 = 5^2+4 \quad t = 5 \quad a = 4$		$D = 53 = 7^2+4 \quad t = 7 \quad a = 4$	
$v_2 = 5$	$+4\cdot 4^0 + 29\cdot 5^2 = 27^2$	$v_2 = 7$	$+4\cdot 4^0 + 53\cdot 7^2 = 51^2$
$p_0 = 2$	$-4\cdot 4^1 + 29\cdot 2^2 = 10^2$	$p_0 = 2$	$-4\cdot 4^1 + 53\cdot 2^2 = 14^2$
$p_1 = 0$	$+4\cdot 4^2 + 29\cdot 0^2 = 8^2$	$p_1 = 0$	$+4\cdot 4^2 + 53\cdot 0^2 = 8^2$
$p_2 = 8$	$-4\cdot 4^3 + 29\cdot 8^2 = 40^2$	$p_2 = 8$	$-4\cdot 4^3 + 53\cdot 8^2 = 56^2$
$p_3 = -80$	$+4\cdot 4^4 + 29(-80)^2 = 432^2$	$p_3 = -1126$	$+4\cdot 4^4 + 53(-112)^2 = 816^2$
$p_4 = 832$	$-4\cdot 4^5 + 29\cdot 832^2 = 4480^2$	$p_4 = 1600$	$-4\cdot 4^5 + 53\cdot 1600^2 = 11648^2$
$p_5 = -8640$	$+4\cdot 4^6 + 29(-8640)^2 = 46528^2$	$p_5 = -22848$	$+4\cdot 4^6 + 53(-22848)^2 = 166336^2$

$v_2 = \frac{t^2+1}{2} \qquad p_0 = t$

(27)

$p_1 = av_2 - 2tp_0 = 4\,\frac{t^2+1}{2} - 2t\cdot t = 2$

$p_2 = ap_0 - 2tp_1 = 4t - 2t\cdot 2 = 0$

$p_3 = ap_1 - 2tp_2 = 4\cdot 2 - 2t\cdot 0 = 8$

$p_4 = ap_2 - 2tp_3 = 4\cdot 0 - 2t\cdot 8 = -16t$

$p_5 = ap_3 - 2tp_4 = 4\cdot 8 - 2t(-16t) = 32(t^2+1)$

przykłady

$D = 29 = 5^2+4 \quad t = 5 \quad a = 4$		$D = 53 = 7^2+4 \quad t = 7 \quad a = 4$	
$v_2 = 13$	$-1\cdot 4^0 + 29\cdot 13^2 = 70^2$	$v_2 = 25$	$-1\cdot 4^0 + 53\cdot 25^2 = 182^2$
$p_0 = 5$	$+1\cdot 4^1 + 29\cdot 5^2 = 27^2$	$p_0 = 7$	$+1\cdot 4^1 + 53\cdot 7^2 = 51^2$

$p_1 = 2$	$-1\cdot 4^2 + 29\cdot 2^2 = 10^2$	$p_1 = 2$	$-1\cdot 4^2 + 53\cdot 2^2 = 14^2$
$p_2 = 0$	$+1\cdot 4^3 + 29\cdot 0^2 = 8^2$	$p_2 = 0$	$+1\cdot 4^3 + 53\cdot 0^2 = 8^2$
$p_3 = 8$	$-1\cdot 4^4 + 29\cdot 8^2 = 40^2$	$p_3 = 8$	$-1\cdot 4^4 + 53\cdot 8^2 = 56^2$
$p_4 = -80$	$+1\cdot 4^5 + 29(-80)^2 = 432^2$	$p_4 = -112$	$+1\cdot 4^5 + 53(-112)^2 = 816^2$
$p_5 = 832$	$-1\cdot 4^6 + 29\cdot 832^2 = 4480^2$	$p_5 = 1600$	$-1\cdot 4^6 + 53\cdot 1600^2 = 11648^2$

$v_2 = t^2+1$ $\qquad$ $p_0 = 2t$

(27)

$$p_1 = av_2 - 2tp_0 = 4(t^2+1) - 2t\cdot 2t = 4$$
$$p_2 = ap_0 - 2tp_1 = 4\cdot 2t - 2t\cdot 4 = 0$$
$$p_3 = ap_1 - 2tp_2 = 4\cdot 4 - 2t\cdot 0 = 16$$
$$p_4 = ap_2 - 2tp_3 = 4\cdot 0 - 2t\cdot 16 = -32t$$
$$p_5 = ap_3 - 2tp_4 = 4\cdot 16 - 2t(-32t) = 64(t^2+1)$$

przykłady

$D = 29 = 5^2+4$ $\quad t = 5$ $\quad a = 4$		$D = 53 = 7^2+4$ $\quad t = 7$ $\quad a = 4$	
$v_2 = 26$	$-4\cdot 4^0 + 29\cdot 26^2 = 140^2$	$v_2 = 50$	$-4\cdot 4^0 + 53\cdot 50^2 = 364^2$
$p_0 = 10$	$+4\cdot 4^1 + 29\cdot 10^2 = 54^2$	$p_0 = 14$	$+4\cdot 4^1 + 53\cdot 14^2 = 102^2$
$p_1 = 4$	$-4\cdot 4^2 + 29\cdot 4^2 = 20^2$	$p_1 = 4$	$-4\cdot 4^2 + 53\cdot 4^2 = 28^2$
$p_2 = 0$	$+4\cdot 4^3 + 29\cdot 0^2 = 16^2$	$p_2 = 0$	$+4\cdot 4^3 + 53\cdot 0^2 = 16^2$
$p_3 = 16$	$-4\cdot 4^4 + 29\cdot 16^2 = 80^2$	$p_3 = 16$	$-4\cdot 4^4 + 53\cdot 16^2 = 112^2$
$p_4 = -160$	$+4\cdot 4^5 + 29(-160)^2 = 864^2$	$p_4 = -224$	$+4\cdot 4^5 + 53(-224)^2 = 1632^2$
$p_5 = 1664$	$-4\cdot 4^6 + 29\cdot 1664^2 = 8960^2$	$p_5 = 3200$	$-4\cdot 4^6 + 53\cdot 3200^2 = 23296^2$

RGXI

$D = t^2 - \frac{4t}{n}$ $\qquad v_2 = n$ $\qquad a = \frac{4t}{n}$ $\qquad$ znak "–"

(19) $\quad p_0^2 - 2tv_2p_0 - (4 - av_2^2) = 0 \qquad p_0^2 - 2tnp_0 - \left(4 - \frac{4t}{n}n^2\right) = 0 \qquad \sqrt{\triangle} = 2(tn-2)$

$$p_0 = \frac{2tn + 2(tn-2)}{2} = 2(tn-1) \qquad p_0 = \frac{2tn - 2(tn-2)}{2} = 2$$

Mamy więc:

$v_2 = n$ $\qquad$ $p_0 = 2$

(28)

$$p_1 = 2tp_0 - av_2 = 2t\cdot 2 - \frac{4t}{n}n = 0$$
$$p_2 = 2tp_1 - ap_0 = 2t\cdot 0 - \frac{4t}{n}\cdot 2 = -\frac{8t}{n}$$
$$p_3 = 2tp_2 - ap_1 = 2t\left(-\frac{8t}{n}\right) - \frac{4t}{n}\cdot 0 = -\frac{16t^2}{n}$$
$$p_4 = 2tp_3 - ap_2 = 2t\left(-\frac{16t^2}{n}\right) - \frac{4t}{n}\left(-\frac{8t}{n}\right) = -\frac{32t^2}{n^2}(tn-1)$$
$$p_5 = 2tp_4 - ap_3 = 2t\left[-\frac{32t^2}{n^2}(tn-1)\right] - \frac{4t}{n}\left(-\frac{16t^2}{n}\right) = -\frac{64t^3}{n^2}(tn-2)$$

przykłady

$D = 128 = 12^2 - \frac{4\cdot 12}{3}$ $t = 12$ $a = 16$ $n = 3$

$v_2 = 3$	$4\cdot 16^0 + 128\cdot 3^2 = 34^2$
$p_0 = 2$	$4\cdot 16^1 + 128\cdot 2^2 = 24^2$
$p_1 = 0$	$4\cdot 16^2 + 128\cdot 0^2 = 32^2$
$p_2 = -32$	$4\cdot 16^3 + 128(-32)^2 = 384^2$
$p_3 = -768$	$4\cdot 16^4 + 128(-768)^2 = 8704^2$
$p_4 = -17920$	$4\cdot 16^5 + 128(-17920)^2 = 202752^2$
$p_5 = -417792$	$4\cdot 16^6 + 128(-417792)^2 = 4726784^2$

$D = 205 = 15^2 - \frac{4\cdot 15}{3}$ $t = 15$ $a = 20$ $n = 3$

$v_2 = 3$	$4\cdot 20^0 + 205\cdot 3^2 = 43^2$
$p_0 = 2$	$4\cdot 20^1 + 205\cdot 2^2 = 30^2$
$p_1 = 0$	$4\cdot 20^2 + 205\cdot 0^2 = 40^2$
$p_2 = -40$	$4\cdot 20^3 + 205(-40)^2 = 600^2$
$p_3 = -1200$	$4\cdot 20^4 + 205(-1200)^2 = 17200^2$
$p_4 = -35200$	$4\cdot 20^5 + 205(-35200)^2 = 504000^2$
$p_5 = -1032000$	$4\cdot 20^6 + 205(-1032000)^2 = 14776000^2$

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RGXI

$D = t^2 + \frac{4t}{n}$ $v_2 = n$ $a = \frac{4t}{n}$ znak "+"

(13) $p_0^2 + 2tv_2p_0 - (4 + av_2^2) = 0$ $p_0^2 + 2tnp_0 - \left(4 + \frac{4t}{n}n^2\right) = 0$ $\sqrt{\triangle} = 2(tn+2)$

$$p_0 = \frac{-2tn + 2(tn+2)}{2} = 2 \qquad p_0 = \frac{-2tn - 2(tn+2)}{2} = -2(tn+1)$$

Mamy więc:

$v_2 = n$ $p_0 = 2$

(27)

$$p_1 = av_2 - 2tp_0 = \frac{4t}{n}\cdot n - 2t\cdot 2 = 0$$

$$p_2 = ap_0 - 2tp_1 = \frac{4t}{n}\cdot 2 - 2t\cdot 0 = \frac{8t}{n}$$

$$p_3 = ap_1 - 2tp_2 = \frac{4t}{n}\cdot 0 - 2t\frac{8t}{n} = -\frac{16t^2}{n}$$

$$p_4 = ap_2 - 2tp_3 = \frac{4t}{n}\cdot\frac{8t}{n} - 2t\left(-\frac{16t^2}{n}\right) = \frac{32t^2}{n^2}(tn+1)$$

$$p_5 = ap_3 - 2tp_4 = \frac{4t}{n}\left(-\frac{16t^2}{n}\right) - 2t\frac{32t^2}{n^2}(tn+1) = -\frac{64t^3}{n^2}(tn+2)$$

przykłady

$D = 160 = 12^2 + \frac{4\cdot 12}{3}$ $t = 12$ $a = 16$ $n = 3$

$v_2 = 3$	$+4\cdot 16^0 + 160\cdot 3^2 = 38^2$
$p_0 = 2$	$-4\cdot 16^1 + 160\cdot 2^2 = 24^2$
$p_1 = 0$	$+4\cdot 16^2 + 160\cdot 0^2 = 32^2$
$p_2 = 32$	$-4\cdot 16^3 + 160\cdot 32^2 = 384^2$
$p_3 = -768$	$+4\cdot 16^4 + 160(-768)^2 = 9728^2$
$p_4 = 18944$	$-4\cdot 16^5 + 160\cdot 18944^2 = 239616^2$
$p_5 = -466944$	$+4\cdot 16^6 + 160(-466944)^2 = 5906432^2$

$D = 245 = 15^2 + \frac{4\cdot 15}{3}$ $t = 15$ $a = 20$ $n = 3$

$v_2 = 3$	$+4\cdot 20^0 + 245\cdot 3^2 = 47^2$
$p_0 = 2$	$-4\cdot 20^1 + 245\cdot 2^2 = 30^2$
$p_1 = 0$	$+4\cdot 20^2 + 245\cdot 0^2 = 40^2$
$p_2 = 40$	$-4\cdot 20^3 + 245\cdot 40^2 = 600^2$
$p_3 = -1200$	$+4\cdot 20^4 + 245(-1200)^2 = 18800^2$
$p_4 = 36800$	$-4\cdot 20^5 + 245\cdot 36800^2 = 576000^2$
$p_5 = -1128000$	$+4\cdot 20^6 + 245(-1128000)^2 = 17656000^2$

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Na następnej stronie przedstawiono wyniki z tego podrozdziału.

Rozwiązania główne RG								
D	v_2	p_0	p_1	p_2	p_3	p_4	p_5	gr.
$(2t_1+1)^2+4$	$2t_1+1=t$ +	− 2	+ 0	− 8	+ $-16t$	− $32(t^2+1)$	+ $-64t(t^2+2)$	VIII
$(2t_1+1)^2+4$	$2t_1^2+2t_1+1=$ $=\frac{t^2+1}{2}$ +	− t	+ 2	− 0	+ 8	− $-16t$	+ $32(t^2+1)$	VIII
$(2t_1+1)^2+4$	$2(2t_1^2+2t_1+1)=$ $=t^2+1$ +	+ $2t$	− 4	+ 0	− 16	+ $-32t$	− $64(t^2+1)$	VIII
$t^2-\frac{4t}{n}$	n −	+ 2	+ 0	+ $-\frac{8t}{n}$	+ $-\frac{16t^2}{n}$	+ $-\frac{32t^2}{n^2}(tn-1)$	+ $-\frac{64t^3}{n^2}(tn-2)$	XI
$t^2+\frac{4t}{n}$	n +	+ 2	+ 0	+ $\frac{8t}{n}$	+ $-\frac{16t^2}{n}$	+ $\frac{32t^2}{n^2}(tn+1)$	+ $\frac{64t^3}{n^2}(tn+2)$	XI